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ON *P*-EXCHANGE RINGS

HIKOJI KAMBARA AND KIYOICHI OSHIRO

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There is a problem concerning the exchange property: which ring R satisfies the condition that every projective right R-module satisfies the exchange property. A ring R with the above condition is said to be a right P-exchange ring. P-exchange rings have been studied in [2], [3], [4], [6], [7], [11], [12], and recently in [9]. Among others, it is shown in [9] that semi-regular rings with right T-nilpotent Jacobson radical are right P-exchange rings, and the converse holds for commutative rings but not in general. It is still open to determine the structure of P-exchange rings. Our main object of this paper is to show that a ring is a right P-exchange ring if and only if all Pierce stalks R_x are right P-exchange rings.

1. Preliminaries

Throughout this paper, all rings R considered are associative and all R-modules are unitary. For an R-module M, $J(M)$ denotes the Jacobson radical of M. For a ring R, $B(R)$ represents the Boolean ring consisting of all central idempotents of R and, as usual, $\mathrm{Spec}(B(R))$ denotes the spectrum of all prime (=maximal) ideals of $B(R)$. For a right R-module M and an element a in M and x in $\mathrm{Spec}(B(R))$ we put $M_x=M/Mx$ and $a_x=a+Mx(\in M_x)$. M_x is called the Pierce stalk of M for x ([8]). Note that $M_x=M\otimes_R R_x$ and R_x is flat as an R-module, hence for a submodule N of M, $N_x\subseteq M_x$. For e in $B(R)$, note that $e_x=1_x$ if and only if $e\in B(R)-x$. Let A and B be right R-modules and x in $\mathrm{Spec}(B(R))$. Then there exists a canonical homomorphism σ from $\mathrm{Hom}_R(A, B)$ to $\mathrm{Hom}_{R_x}(A_x, B_x)$. We denote $f^x=\sigma(f)$ for f in $\mathrm{Hom}_R(A, B)$. We note that if A is projective, then σ is an epimorphism.

We will use later the following well known facts [8]:

a) Let M and N be finitely generated right R-modules with $M\subseteq N$. If $x\in\mathrm{Spec}(B(R))$ and $M_x=N_x$ then $Me=Ne$ for suitable e in $B(R)-x$.

b) For right R-modules M and N with $M\supseteq N$, if $N_x=M_x$ for all x in $\mathrm{Spec}(B(R))$, then $M=N$.

c) A ring R is a commutative reguler ring if and only if all stalks R_x are fields, and similarly, a ring R is a strongly reguler ring if and only if all R_x are division rings.