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SEQUENCES OF DERIVABLE TRANSLATION PLANES

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1. Introduction

In [6], Hiramine, Matsumoto, and Oyama study translation planes of order q^4 and kernel $\supseteq K \cong GF(q^2)$ that admit an affine elation group E of order at least q^2 and a Baer group $\mathcal{B}$ of order $q+1$ such that $[E, \mathcal{B}] \neq 1$. It is shown in [6] that for q odd, such translation planes always have solvable translation complements. Further, a construction is given by which translation planes of the above type may be obtained from arbitrary translation planes of order h^2 and kernel $\supseteq GF(h)$.

In this article, we extend the construction method of Hiramine, Matsumoto, and Oyama and show how to obtain infinite sequences of potentially new derivable translation planes. This method allows the identification of certain other recently constructed translation planes.

In [2], Boerner-Lantz constructs a new class of semifield planes of order q^4. We show that these planes may be obtained by the construction methods under consideration from the Desarguesian planes. Furthermore, there are other similar but nonisomorphic semifield planes which may be constructed from Desarguesian planes.

We also complete the study of the groups of the translation planes of order q^4, kernel $GF(q^2)$ admitting elation and Baer groups as described above for q even.

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2. The collineation groups

In this section, we extend the results of Hiramine, Matsumoto, and Oyama to include the even order case.

Theorem 2.1 (See Hiramine, et al. [6] for q odd.). *Let π denote a translation plane of order q^4 and kernel $\geq K \cong GF(q^2)$ which admits an elation group E of order $\geq q^2$ and a Baer group $\mathcal{B}$ of order $q+1$ such that $[\mathcal{B}, E] \neq 1$. Then the full collineation group is solvable.*