Mukerjee, R., Jimbo, M. and Kageyama, S.
Osaka J. Math.
24 (1987), 395–407

ON CYCLIC SEMI-REGULAR GROUP DIVISIBLE DESIGNS

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(Received February 25, 1986)

1. Introduction

The group divisible (GD) designs constitute the largest, simplest and perhaps most important type of 2-associate partially balanced incomplete block (PBIB) designs. A *GD design* is an arrangement of v ($=mn$) treatments in b blocks such that each block contains k ($<v$) distinct treatments; each treatment is replicated r times; and the set of treatments can be partitioned into m ($\geqq 2$) equivalence classes of n ($\geqq 2$) treatments each, any two distinct treatments occurring together in λ_1 blocks if they belong to the same equivalence class, and in λ_2 blocks if they belong to different equivalence classes. It may be remarked that in the literature the commonly used terminology for these equivalence classes of treatments is "groups", but here we deliberately prefer to use the phrase "equivalence classes" in order to avoid a notational confusion with groups in a group-theoretic sense which we shall be considering shortly in this paper. GD designs may again be of three types: (a) *singular*, if $r=\lambda_1$; (b) *semi-regular* (SR), if $r>\lambda_1$ and $rk=\lambda_2 v$; (c) *regular* (R), if $r>\lambda_1$ and $rk>\lambda_2 v$.

If the automorphism group of a GD design contains a cyclic group of order v, then the GD design is said to be *cyclic*. For a cyclic GD design, without loss of generality, we may represent the set of v treatments by $V=\{0, 1, \cdots, v-1\}$ and in this case the automorphism of order v is $x \rightarrow x+1 \pmod{v}$. In the sequel, we shall use this notation to represent the treatments in a cyclic GD design. The following definitions will also be helpful. For a block $B=\{b_0, b_1, \cdots, b_{k-1}\}$ and any $i \in V$, define $B+i=\{b_0+i, b_1+i, \cdots, b_{k-1}+i\}$, addition being reduced mod v. The collection of blocks $\{B+i \mid i \in V\}$ is called the *full orbit* containing B. Let i_0 be the smallest positive integer such that $B+i_0=B$. If $i_0<v$, then the collection of blocks $\{B+i \mid 0 \leqq i \leqq i_0-1\}$ is called a *short orbit* containing B.

A large number of methods of constructing GD designs are available in the literature (cf. Clatworthy [4], Raghavarao [13]). However, most of the designs produced by them are not cyclic. Cyclic GD designs can be conveniently obtained by the method of differences of Bose [1]. Their flexibility, ease of representation and conduct of experimentation make them worthy of