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A TOPOLOGICAL INVARIANT RELATED TO THE NUMBER OF ORTHOGONAL GEODESIC CHORDS

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1. Introduction

A geodesic on a Riemannian manifold with boundary is called an *orthogonal geodesic chord* if it connects two points of the boundary and intersects with the boundary orthogonally at both end points. For orthogonal geodesic chords, Lyusternik and Schnirelmann [7] prove the existence of n such chords of any convex body in $\boldsymbol{R}^n$ (compact convex C^∞ submanifold with boundary of $\boldsymbol{R}^n$ with an interior point) and Bos [1] extends it to locally convex disks of dimension n (the precise definition is given later).

In this note, we denote by M a compact Riemannian C^∞ manifold of dimension n with boundary, define a topological invariant integer $\nu(M)$ and show that there exist at least $\nu(M)$ non-constant orthogonal geodesic chords of M if the boundary is *locally convex* with respect to the Riemannian metric given on M, namely if there exists a positive number η such that for any two points p and q of the boundary with $d(p, q)<\eta$, where d is the distance derived from the Riemannian metric, there is the unique geodesic in M w.r.t. the Riemannian metric, connecting the two points p and q. Furthermore we show $\nu(D^n)=n$, where D^n is the n dimensional disk, and

$$\nu(M)\geq n$$

if M is contractible ($n=dim\ M$).

For $n=1$ and 2, we know

(*) "compact contractible manifold with boundary is always homeomorphic to a disk."

For $n=3$, this statement is equivalent to the Poincaré Conjecture, that is, we have (*) iff the conjecture for three dimension is true. For $n\geq 4$, there are examples of compact contractible manifolds with boundary, which are not homeomorphic to a disk [8]. As a corollary, we have a generalization of Bos'