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# ETALE ENDOMORPHISMS OF ALGEBRAIC VARIETIES

Dedicated to Professor Minoru Nakaoka on his 60th birthday

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## 1. Introduction

Let $k$ be an algebraically closed field of characteristic zero, which we fix as the ground field throughout this article. Let $f$: $X \to X$ be an étale endomorphism of an algebraic variety $X$. Then $f$ is, in particular, a quasi-finite morphism. We shall be concerned with the following:

PROBLEM. *Is an étale endomorphism $f$: $X \to X$ finite?*

If $f$ is set-theoretically injective then $f$ is bijective by Ax's theorem [1, 3]; hence $f$ is an automorphism. If $X$ is complete, $f$ is clearly finite. In the case where $X$ is the affine $n$-space $\boldsymbol{A}_k^n$, the Jacobian conjecture (cf. [2]) is equivalent to showing that $f$: $X \to X$ is finite. In the following *we assume that $X$ is a non-singular, non-complete algebraic variety.* Our results show that $f$ is an automorphism (hence finite) for a fairly wide class of varieties $X$, while there are abundant examples of varieties $X$ with non-finite étale endomorphisms.

## 2. Preliminary result

We recall the logarithmic ramification formula (cf. Iitaka [6]). Let $f$: $X \to Y$ be a dominant morphism of nonsingular algebraic varieties. Then there exist nonsingular complete varieties $V$ and $W$ and a dominant morphism $\phi$: $V \to W$ satisfying the following conditions:

(1) $X$ and $Y$ are open subsets of $V$ and $W$, respectively; hence $V$ and $W$ are nonsingular completions of $X$ and $Y$, respectively;

(2) the boundaries $D := V - X$ and $\Delta := W - Y$ are the divisors with simple normal crossings; namely, all irreducible components of $D$ (or $\Delta$) are nonsingular subvarieties of codimension 1 intersecting each other normally at every point of intersection of $D$ (or $\Delta$); we denote by the symbol $D$ (or $\Delta$) the reduced divisor whose support is $D$ (or $\Delta$);

(3) the restriction of $\phi$ onto $X$ coincides with $f$; hence $\phi^{-1}(\Delta) \subseteq D$.

Denote by $K_V$ (or $K_W$) the canonical divisor of $V$ (or $W$). The logarithmic ramification formula then asserts that there exists an effective divisor $R_\phi$ such that