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KLEIN BOTTLES IN GENUS TWO 3-MANIFOLDS

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Introduction

For a closed 3-manifold M, it is very interesting to study the relation between a Heegaard surface of M and an embedded surface in M. For this purpose W. Haken has shown in [2] that if a closed 3-manifold M is not irreducible, then there is an essential 2-sphere in M which intersects a fixed Heegaard surface of M in a single circle, and W. Jaco has given in [4] an alternative proof of it. M. Ochiai has shown in [8] that if a closed 3-manifold M contains a 2-sided projective plane, then there is a 2-sided projective plane in M which intersects a fixed Heegaard surface of M in a single circle, and moreover he has shown in [9] that if a closed 3-manifold M with a Heegaard splitting of genus two contains a 2-sided projective plane, then M is homeomorphic to $P^2\times S^1$. Succesively T. Kobayashi has shown in [5] that if a closed 3-manifold M with a Heegaard splitting of genus two contains a 2-sided non-separating incompressible torus, then there is a 2-sided non-separating incompressible torus in M which intersects a fixed Heegaard surface in a single circle. In this paper we will show a similar result for a Klein bottle.

Theorem 1. *Let M be a closed connected orientable 3-manifold with a fixed Heegaard splitting $(V_1, V_2; F)$ of genus two. If M contains a Klein bottle, then there is a Klein bottle in M which intersects F in a single circle.*

By the way it is well known that a closed orientable 3-manifold M with a Heegaard splitting of genus one contains a Klein bottle if and only if M is homeomorphic to $L(4n, 2n+1)$ for some non-negative integer n (c.f. [1]). Using Theorem 1 we will give a necessary and sufficient condition for a closed orientable 3-manifold with a Heegaard splitting of genus two to contain a Klein bottle. Namely we will give three families of closed orientable 3-manifolds, and we will show that a closed orientable 3-manifold M with a Heegaard splitting of genus two contains a Klein bottle if and only if M belongs to one of the three families (Theorem 2).

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