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AUTOMORPHISMS OF p-GROUPS OF SEMIFIELD TYPE

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1. Introduction

Let $\pi=\pi(D)$ be a finite projective plane coordinatized by a semifield D of order q. Let A be the collineation group of all elations with axis $[\infty]$ and B the collineation group of all elations with center (∞). We denote by $P(\pi)$ the collineation group generated by A and B. Set $P=P(\pi)$. Then P has the following properties:

(i) $P=AB$, $|P|=q^3$, where q is a power of a prime p, and A and B are elementary abelian normal subgroups of P of order q^2.

(ii) $ab=ba$ implies $a\in A\cap B$ or $b\in A\cap B$ for all $a\in A$ and $b\in B$.

A p-group P is called a p-group of semifield type if it satisfies (i) and (ii) as above. This is the same as a T-group satisfying that all $a\in A-A\cap B$ and all $b\in B-A\cap B$ are regular, defined in [1].

In the paper [1], A. Cronheim has proved as part of a more general theorem that a finite semifield can be constructed for the group P and the ordered pair (A, B). We denote the semifield by $D(A, B)$ and the set of all such ordered pairs (A, B) by V_P. Let W_P denote the set of all abelian subgroups of P of order q^2. Then one of the following holds (Lemma 4.1).

(i) $p=2$ and $|V_P|=2$.

(ii) $p>2$ and $V_P=\{(A, B)\,|\,A\neq B,\ A, B\in W_P\}$.

In this paper we will study the semifields constructed for all (A, B) in V_P.

Let (A, B) and (A', B') be elements in V_P. Then $D(A, B)$ and $D(A', B')$ are isotopic if and only if there exists an automorphism f of P which maps A onto A' and B onto B' (Lemma 4.2). Therefore, we will consider the action of $\mathrm{Aut}(P)$ on the set W_P and will prove the following.

Theorem 4.8. *Let P be a p-group of semifield type of order p^{3n} for an odd prime p and a positive integer n and assume $|W_P|>2$. Set $L=\mathrm{Aut}(P)$, $G=C_L(Z(P))$ and $W=W_P$. Then*

(i) $|W|=1+p^r$ *for a positive divisor r of n.*