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PROJECTIVE MODULES OVER VON NEUMANN REGULAR RINGS HAVE THE FINITE EXCHANGE PROPERTY

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It is an open problem to determine which ring satisfies the condition that every projective module over the ring has the (finite) exchange property.

For this problem, Harada and Ishii ([1]) and Yamagata ([5]) have shown that projective modules over perfect rings have the exchange property, and recently Kutami and Oshiro ([3]) have shown that projective modules over a certain Boolean ring have the exchange property.

The purpose of the present note is to show that projective modules over (von Neumann) regular rings have the finite exchange property.

Throughout this paper we assume a ring R has identity and all R-modules are unitary. A right R-module M has the *exchange property* if for any right R-module X and any two decompositions:

$$X = M'\oplus N = \sum_I \oplus A_\alpha$$

with $M' \simeq M$, there exist submodules $A'_\alpha \subseteq A_\alpha$ such that

$$X = M'\oplus(\sum_I\oplus A'_\alpha)\,.$$

M has the *finite exchange property* if this holds whenever the index set I is finite.

For a given projective right module P, the following conditions due to Nicholson [4] are useful in the study of the exchange property:

(C_1): If $P=P_1+P_2$ where P_i are submodules there exists a submodule $P'_i\subseteq P_i$ for $i=1, 2$ such that

$$P = P'_1\oplus P'_2\,.$$

(C_2): If $P=\sum_I P_\alpha$ where P_α are submodules there exists a decomposition

$$P = \sum_I\oplus P'_\alpha$$

with $P'_\alpha\subseteq P_\alpha$ for each $\alpha\in I$.

Nicholson ([4]) has shown that a projective right R-module P has the