Imayoshi, Y.
Osaka J. Math.
20 (1983), 581–598

UNIVERSAL COVERING SPACES OF CERTAIN QUASI-PROJECTIVE ALGEBRAIC SURFACES

YOICHI IMAYOSHI

(Received October 23, 1981)

Introduction. In this paper we investigate some function-theoretic properties of universal covering spaces of certain quasi-projective algebraic surfaces.

Let $\hat{X}$ be a two-dimensional complex manifold and let C be a one-dimensional analytic subset of $\hat{X}$ or an empty set. Let R be a Riemann surface. We assume that a proper holomorphic mapping $\hat{\pi}$: $\hat{X}\rightarrow R$ satisfies the following two conditions: (i) $\hat{\pi}$ is of maximal rank at every point of $\hat{X}$, and (ii) by setting $X=\hat{X}-C$ and $\pi=\hat{\pi}|X$, the fiber $S_p=\pi^{-1}(p)$ over each point p of R is an non-singular irreducible analytic subset of X and is of fixed finite type (g, n) with $2g-2+n>0$ as a Riemann surface, where g is the genus of S_p and n is the number of punctures of S_p. We call such a triple (X, π, R) a holomorphic family of Riemann surfaces of type $(,g\ n)$ over R. We also say that X has a holomorphic fibration (X, π, R) of type (g, n).

We assume throughout this paper R is a non-compact Riemann surface of finite type and its universal covering space is the unit disc $D=(|t|<1)$ in the complex t-plane.

P.A. Griffiths [2] got the following uniformization theorem of quasi-projective algebraic surfaces. Let $\hat{X}$ be a two-dimensional, irreducible, smooth, quasi-projective algebraic varitey over the complex numbers. Then for every point x in $\hat{X}$, there exists a Zariski neighborhood X of x in $\hat{X}$ such that X has a holomorphic fibration (X, π, R) as above. Then the universal covering space $\tilde{X}$ of X is topologically a cell. Griffiths proved that $\tilde{X}$ is biholomorphically equivalent to a bounded domain of holomorphy in C^2 using the theory of simultaneous uniformization of Riemann surfaces due to Bers. (cf. Bers [1].) The function-theoretic properties of such interesting domains $\tilde{X}$ are little studied. (cf. Shabat [10].)

At the begining, in § 1, we recall some notations and results of [3], [4] and [5] which will be used later. Let $\mathcal{M}$ be the homotopic monodromy group of (X, π, R), which will be defined in § 1. Then we get the following theorems in § 2, § 3, § 4 and § 5.