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EQUIVARIANT IMMERSIONS AND EMBEDDINGS OF SMOOTH G-MANIFOLDS

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Introduction

Let $f: P \to Q$ be a smooth G-map between smooth G-manifolds P, Q where G is a compact Lie group. The aim of this paper is to give sufficient conditions for f to be equivariantly homotopic to an orthogonally isovariant map, an isovariant immersion, and an equivariant embedding. We work in the smooth category. In the PL category with G finite there is a result of Illman [3]. In the smooth category, when Q is a euclidean representation space, there are also results of Wasserman [6], Marchow and Pulikowski [4]. As Illman pointed out in his paper the existence of equivariant PL and equivariant smooth embeddings are questions of completely different natures. In the PL category, conditions for the dimensions of fixed point sets of each subgroup of G give a sufficient condition for the existence of equivariant embeddings. In the smooth category, however, those are not sufficient. Conditions also for the normal representations around fixed point sets are needed.

Let G_x denote the isotropy subgroup of G at $x \in P$. A G-map $f: P \to Q$ is called *isovariant* if $G_x = G_{f(x)}$ for every $x \in P$. Let $\mathcal{I}(P)$ denote the set of isotropy types (i.e., conjugacy classes of isotropy subgroups) on P. For any $(H) \in \mathcal{I}(P)$ define

$$P^{(H)} = \{x \in P \mid H \text{ is conjugate to a subgroup of } G_x\},$$
$$P_{(H)} = \{x \in P \mid H \text{ is conjugate to } G_x\}.$$

$\nu(P_{(H)})$ denotes the normal bundle of $P_{(H)}$ in P. The differential df of a smooth isovariant map $f: P \to Q$ induces a bundle homomorphism

$$\tilde{d}f: \nu(P_{(H)}) \to \nu(Q_{(H)}).$$

If, for every $(H) \in \mathcal{I}(P)$, $\tilde{d}f$ is a bundle monomorphism (i.e., a monomorphism on each fibre), f is called *orthogonally isovariant*.

Our main results are Theorem 7.2, Corollary 7.3 and Corollary 7.4 stated in section 7. Theorem 7.2 shows that, under conditions for the dimensions of fixed point sets and also for the normal representations around them, a smooth