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SUFFICIENCY AND PAIRWISE SUFFICIENCY IN STANDARD BOREL SPACES-II

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Introduction

Let $(X, \boldsymbol{A}, P_\theta\colon \theta\in\Theta)$ be an experiment and $\boldsymbol{B}$ a sub σ-algebra of $\boldsymbol{A}$. It is known and can be proved easily [9], that if $\{P_\theta\colon \theta\in\Theta\}$ is dominated by a σ-finite measure then pairwise sufficiency of $\boldsymbol{B}$ implies its sufficiency. There has been attempts to generalise this result and show that even in the undominated case paiwise sufficiency is related to sufficiency. Pitcher [11] introduced compact statistical structures, Basu and Ghosh [1] discrete statitiscal structures and finally Hasegawa and Perlman [6] coherent experiments. It is now known that [4] coherence is equivalent to compactness and the discrete structure a special case of both. That these concepts are natural generalisation of domination was established by Dipenbrock [3], who showed that compactness and coherence are both equivalent to domination by a localizable measure. Their theorems connecting pairwise sufficiency with sufficiency is of the form "if $\boldsymbol{B}$ is pairwise sufficient then $\bigcap_{\theta_1,\theta_2} \boldsymbol{B}\vee N_{\theta_1,\theta_2}$ is sufficient".

While experiments dominated by a σ-finite measure are coherent, Rogge [13] showed that if $\boldsymbol{A}$ is countably generated then any coherent experiment is necessarily dominated by a σ-finite measure. Thus in countably generated situations' in particular in the Standard Borel Case, compactness is not more general than domination by a σ-finite measure. However it is proved in [12] that, in the Standard Borel case if P_θ's are discrete then pairwise sufficiency is equivalent to sufficiency. Since $P_\theta(x)$ can be thought of as density with respect to the counting measure, a similar generalisation seems possible. This paper centres on such a generalisation.

This paper is motivated by the work of Hasegawa.—Perlman and the theorem of Dipenbrock. We define the notion of weak coherence, Borel localizable and Borel decomposable measures—all standard Borel adaptations of known concepts. It is then shown that experiments dominated by a Borel localizable measure satisfying an additional measurability condition are weakly coherent. For weakly coherent experinents we show that if $\boldsymbol{B}$ is countably generated and pairwise sufficient then $\bigcap_{\theta_1,\theta_2} \boldsymbol{B}\vee N_{\theta_1,\theta_2}$ is sufficient.