

NORMAL HOMOGENEOUS METRICS AND THEIR SPECTRA

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Introduction. Let K be a compact Lie group acting almost effectively and transitively on a compact manifold M . A Riemannian metric on M is called K -normal homogeneous if it is induced canonically from a biinvariant metric on K . We are mainly interested in classifying normal homogeneous metrics on compact homogeneous spaces up to homothetical equivalence.

We put forward our study by means of the spectrum and the eigenspaces of the Laplacian by the following reasons: First, a K -normal homogeneous metric has a remarkable property that the eigenvalues of its Laplacian are expressed explicitly in terms of the representation of the Lie group K and moreover the computations may be carried out in a relatively simple manner. Secondly, the spectrum may give information on the Riemannian manifold which may not be obtained by the curvatures. For example, although flat tori have vanishing curvatures, their spectra considerably distinguish the isometry classes of them. We see indeed that if two Riemannian metrics on a compact manifold have the same spectrum and the eigenspaces, then they are identical, as is shown in 1 (Lemma 1.1).

In the paper [2], Berger has shown that certain normal homogeneous metrics on S^n and $P^n(\mathbb{C})$ (n : odd) are not isometric to the usual ones. In 5, we shall give some results for compact irreducible symmetric spaces, extending the above results. In fact, we shall compute certain eigenvalues of the Laplacian and prove our theorem, using the work of Oniřćik on the classification of transitive compact connected transformation groups on compact manifolds. We see then the following: Let $M=K'/L'$ be a compact irreducible symmetric space given by the symmetric pair (K', L') with a compact simple Lie group K' . Let K be a connected closed subgroup of K' which is transitive on M . Then, a K -normal homogeneous metric on M is isometric to the original symmetric metric if and only if the linear isotropy representation of $L=K \cap L'$ is irreducible. Moreover, while there exist several such subgroups of $SO(n+1)$ acting on S^n (n : odd), many of the normal homogeneous metrics of them are mutually homothetically inequivalent.

In 3, we shall consider one parameter families of K -normal homogeneous