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ON THE ALEXANDER POLYNOMIALS OF SLICE LINKS

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The purpose of this note is to generalize the theorem that the Alexander polynomial of a slice knot is of the form $f(t)\cdot f(t^{-1})$ for an integral polynomial $f(t)$ with $|f(1)|=1$ (see [3]). We will show the following:

Theorem. *Let L be a slice link with μ components in the strong sense, then there exists an integral polynomial $F(t_1, \cdots, t_\mu)$ with $|F(1, \cdots, 1)|=1$ and the Alexander polynomial $A(t_1, \cdots, t_\mu)$ of L is of the form*

$$A(t_1, \cdots, t_\mu) \doteq F(t_1, \cdots, t_\mu)\cdot F(t_1^{-1}, \cdots, t_\mu^{-1})^{(*)} .$$

Conversely for a given integral polynomial $F(t_1, \cdots, t_\mu)$ with $|F(1, \cdots, 1)|=1$, there exists a slice link with μ components in the strong sense whose Alexander polynomial is $F(t_1, \cdots, t_\mu)\cdot F(t_1^{-1}, \cdots, t_\mu^{-1})$.

To prove the above Theorem, we will consider two theorems. In §2 the necessary condition of the Alexander polynomials will be considered for not only slice links in the strong sense, but also cobordant links. We will prove the following:

Theorem 1. *For cobordant links L_i, $i=1, 2$, with μ components, there exist two integral polynomials $F_i(t_1, \cdots, t_\mu)$, $i=1, 2$, with $|F_i(1, \cdots, 1)|=1$ such that*

$$\begin{aligned} &A_1(t_1, \cdots, t_\mu)\cdot F_1(t_1, \cdots, t_\mu)\cdot F_1(t_1^{-1}, \cdots, t_\mu^{-1}) \\ &\doteq A_2(t_1, \cdots, t_\mu)\cdot F_2(t_1, \cdots, t_\mu)\cdot F_2(t_1^{-1}, \cdots, t_\mu^{-1}) , \end{aligned}$$

where A_i is the Alexander polynomial of the link L_i.

Since a slice link L with μ components in the strong sense is cobordant to the trivial link with μ components, the following corollary will be obtained.

Corollary. *The Alexander polynomial $A(t_1, \cdots, t_\mu)$ of a slice link L with μ components in the strong sense necessarily satisfies $A(t_1, \cdots, t_\mu) \doteq F(t_1, \cdots, t_\mu)$*

* The notation "$\doteq$" means equal up to $\pm t_1^{n_1} t_2^{n_2} \cdots t_\mu^{n_\mu}$ for suitable integers $n_1, \cdots, n_\mu$.