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THE SYMBOL CALCULUS FOR THE FUNDAMENTAL SOLUTION OF A DEGENERATE PARABOLIC SYSTEM WITH APPLICATIONS

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Introduction

In the paper [2] S. D. Eidelman has constructed the fundamental solution of a system of partial differential operators which is parabolic in the sense of Petrowski with sufficiently smooth coefficients. A few years later, the assumptions on the smoothness of the coefficients have been weakened to uniform Hölder continuity. The bibliography and bibliographical remarks concerning this topics are found in A. Friedman's book [4]. The applications of the fundamental solution to the study of the Cauchy problem and other related problems are found in the above book and S. D. Eidelman's book [3]. On the other hand, if the coefficients are sufficiently smooth, the recent results of the theory of pseudo-differential operators, especially that of H. Kumano-go [8] and [9], have enabled us to construct a symbol of the fundamental solution of a parabolic operator which may be of degenerate type through only the symbol calculus. (See the paper C. Tsutsumi [18].)

In the present paper we shall, using a method similar to that of [18], construct the fundamental solution of a degenerate parabolic system $L=\partial_t+p(t; X, D_x)$ which has the property (F) (See the Definition 2.2). A system of partial differential operators which is parabolic in the sense of Petrowski with C^∞-coefficients has this property, and so do the operators treated in T. Matsuzawa [11], B. Helffer [6], C. Tsutsumi [18] and M. Miyake [12]. In the papers [11], [6] and [12], a family of parametrices $K_0+K_1+\cdots+K_j$ of the operator L is constructed so that they satisfy the equation $L_{t,x}(\sum_{k=0}^{j}K_k(x, y, t, t'))=\delta(x-y, t-t')+F_j(x, y, t, t',)$, and $K_1, \cdots, K_j$ and F_j are very regular. In [18] and the present paper, however, the fundamental solution is constructed in the class of pseudo-differential operators.

In section 1 we shall give some lemmas on the symbol calculus. In section 2 the matrix $e(t, s; x, \xi)$ of symbols of fundamental solution will be constructed and its asymptotic expansion will be given in a very natural form (See the formula (2.23)). In section 3 the general result of section 2 is applied to a