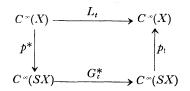
## SPHERICAL MEANS ON RIEMANNIAN MANIFOLDS

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1. Let X be a compact Riemannian manifold of dimension n, TX its tangent bundle and SX its unit sphere bundle. Denote by  $p: SX \rightarrow X$  the canonical projection. Let  $G_t: SX \rightarrow SX(t \in \mathbb{R})$  be the geodesic flow.

The spherical mean (of radius t)  $L_t: C^{\infty}(X) \to C^{\infty}(X)$  is defined by the following commutative diagram:



Here  $p^*$  and  $G_t^*$  denote the maps induced, respectively, by p and  $G_t$ , and  $p_1$  is the fibre integral defined by

$$p_! f(x) = \int_{p^{-1}x} f \omega_F, \qquad f \in C^{\infty}(SX),$$

 $\omega_F$  being the volume element on the fibre of p defined naturally by the Riemannian metric on X.

In this paper we prove the following

**Theorem I.** For sufficiently small positive t,  $L_t$  is a Fourier integral operator of order  $-\frac{1}{2}(n-1)$ , which belongs to the class determined by the conormal bundle  $\Lambda \subset T^*(X \times X) \setminus 0$  of  $\Delta_t = \{(x, y); d(x, y) = t\} \subset X \times X$ , d being the metric induced by the Riemannian metric.

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2. For convenience sake, we consider all the operators as acting on the spaces of half densities. Let  $\Omega_{\frac{1}{2}}(X)$  denote the bundle of half densities on X and  $C^{\infty}\Omega_{\frac{1}{2}}(X)$  the space of smooth cross-sections of  $\Omega_{\frac{1}{2}}(X)$ . The Riemannian metric of X induces canonical densities  $\omega_X$  and  $\omega_{SX}$ , respectively, on X and SX, which allow us to identify  $C^{\infty}(X)$  with  $C^{\infty}\Omega_{\frac{1}{2}}(X)$ ,  $C^{\infty}(SX)$  with  $C^{\infty}\Omega_{\frac{1}{2}}(SX)$ , respectively,