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SPHERICAL MEANS ON RIEMANNIAN MANIFOLDS

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1. Let X be a compact Riemannian manifold of dimension n, TX its tangent bundle and SX its unit sphere bundle. Denote by p: $SX \to X$ the canonical projection. Let G_t: $SX \to SX$ $(t \in \mathbf{R})$ be the geodesic flow.

The spherical mean (of radius t) L_t: $C^\infty(X) \to C^\infty(X)$ is defined by the following commutative diagram:

$$\begin{tikzcd} C^\infty(X) \arrow[r, "L_t"] \arrow[d, "p^*"'] & C^\infty(X) \\ C^\infty(SX) \arrow[r, "G_t^*"] & C^\infty(SX) \arrow[u, "p_!"'] \end{tikzcd}$$

Here p^* and G_t^* denote the maps induced, respectively, by p and G_t, and $p_!$ is the fibre integral defined by

$$p_! f(x) = \int_{p^{-1}x} f\omega_F\,, \qquad f \in C^\infty(SX)\,,$$

ω_F being the volume element on the fibre of p defined naturally by the Riemannian metric on X.

In this paper we prove the following

Theorem I. *For sufficiently small positive t, L_t is a Fourier integral operator of order $-\frac{1}{2}(n-1)$, which belongs to the class determined by the conormal bundle $\Lambda \subset T^*(X \times X) \backslash 0$ of $\Delta_t = \{(x, y);\ d(x, y) = t\} \subset X \times X$, d being the metric induced by the Riemannian metric.*

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2. For convenience sake, we consider all the operators as acting on the spaces of half densities. Let $\Omega_{\frac{1}{2}}(X)$ denote the bundle of half densities on X and $C^\infty\Omega_{\frac{1}{2}}(X)$ the space of smooth cross-sections of $\Omega_{\frac{1}{2}}(X)$. The Riemannian metric of X induces canonical densities ω_X and ω_{SX}, respectively, on X and SX, which allow us to identify $C^\infty(X)$ with $C^\infty\Omega_{\frac{1}{2}}(X)$, $C^\infty(SX)$ with $C^\infty\Omega_{\frac{1}{2}}(SX)$, respectively,