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$\tilde{H}$-COBORDISM, I; THE GROUPS AMONG THREE DIMENSIONAL HOMOLOGY HANDLES

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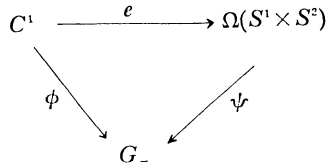
This paper will introduce a concept of a cobordism theory, called $\tilde{H}$-cobordism, between 3-dimensional homology handles. The set of the types of distinguished homology orientable handles modulo $\tilde{H}$-cobordism relation will form an abelian group $\Omega(S^1\times S^2)$, called *the $\tilde{H}$-cobordism group of homology orientable handles*. As a basic property of the $\tilde{H}$-cobordism group $\Omega(S^1\times S^2)$ the following commutative triangle will be established:



Here, C^1 is the Fox-Milnor's 1-knot cobordism group (See Fox-Milnor [3].), G_- is the Levine's integral matrix cobordism group (See Levine [9].), e is a homomorphism and ϕ, ψ are epimorphisms. In particular the $\tilde{H}$-cobordism group $\Omega(S^1\times S^2)$ will have an infinite rank. Analogously *the $\tilde{H}$-cobordism group $\Omega(S^1\times_\tau S^2)$ of homology non-orientable handles* will be also constructed. We shall show that the $\tilde{H}$-cobordism group $\Omega(S^1\times_\tau S^2)$ is isomorphic to the direct sum of infinitely many copies of the cyclic group of order two. Furthermore, it will be shown that the assignment $\tau: m\to m'$ of the type m of any distinguished homology non-orientable handle to the type m' of its 2-fold orientation-cover (which is a distinguished homology orientable handle) induces a well-defined homomorphism $\tau^*: \Omega(S^1\times_\tau S^2)\to T_2\subset\Omega(S^1\times S^2)$ from $\Omega(S^1\times_\tau S^2)$ to the subgroup T_2 of $\Omega(S^1\times S^2)$ consisting of elements of order two. As one consequence T_2 will be infinitely generated.

Section 1 will construct the $\tilde{H}$-cobordism group $\Omega(S^1\times S^2)$ of homology orientable handles. In Section 2 we will discuss the properties of the invariants of $\Omega(S^1\times S^2)$ and compare $\Omega(S^1\times S^2)$ with Fox-Milnor's 1-knot cobordism group C^1 and with the Levine's integral matrix cobordism group G_-. Section 3 will concern the zero element and the order-two-elements of the $\tilde{H}$-cobordism group $\Omega(S^1\times S^2)$. It will be shown that *the type m of a distinguished homology orientable*