Aschbacher, M. and Seitz, G.M.
Osaka J. Math.
13 (1976), 439–482

ON GROUPS WITH A STANDARD COMPONENT OF KNOWN TYPE

MICHAEL ASCHBACHER* AND GRAY M. SEITZ**

(Received June 2, 1975)

1. Introduction and notation

Let G be a finite group containing a standard subgroup of known isomorphism type, centralized by a 4-group. Then it is shown that G is a known group or G is of Conway Type. The proof requires information about the classes of involutions and centralizers in the automorphism groups of the known sporadic groups, and that information is summarized below in tabular form, as it is of independent interest.

The main theorem is a step toward the classification of finite groups of component type. To put the result in the proper setting we include the following definitions and background material.

A group A is *quasisimple* if A is its own commutator group and, modulo its center, A is simple. A component of a group is a subnormal quasisimple subgroup. The *core* of a group is its largest normal subgroup of odd order. *A 2-component* of a group is a subnormal subgroup A such that A is its own commutator group and A is quasisimple modulo its core. G is of *component type* if the centralizer in G of some involution contains a 2-component. This is equivalent to requiring that the centralizer is not 2-constrained.

The following important conjecture of J. G. Thompson seems close to being established:

B-conjecture: Let G be a finite core free group. Then 2-components of centralizers of involutions are quasisimple.

A subgroup K of G is *tightly embedded* in G if K has even order while K intersects its distinct conjugates in subgroups of odd order. A *standard subgroup* of G is a quasisimple subgroup A of G such that $K=C_G(A)$ is tightly embedded in G, $N_G(A)=N_G(K)$, and A commutes with none of its conjugates. It is shown in [1] and [14] that:

Component Theorem. Let G be a finite group of component type satisfying the B-conjecture and contained in the automorphism group of a

* Partial support supplied by the Alfred P. Sloan Foundation and by NSF GP-35678
** Partial support supplied by the National Science Foundation.