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A NOTE ON RELATIVE T-NILPOTENCY

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This note gives some supplementary results of [6]. The first one shows an application of the idea in the proof of [6], Lemma 7 and gives a characterization of artinian rings. The second one gives a refinement of [6], Corollary 2 to Theorem A.2 and the final one is a special type of the exchange property.

Throughout we shall assume that R is a ring with identity and modules are unitary right R-modules. First, we shall recall definitions in [6].

Let $\{P_\alpha\}_I$ and $\{Q_\beta\}_J$ be two infinite sets of R-modules. We take a countable set $\{M_i\}_1^\infty$ such that $M_{2i-1}=P_{\alpha(2i-1)}\in\{P_\alpha\}_I$ and $M_{2j}=Q_{\beta(2j)}\in\{Q_\beta\}_J$. Further we take a set of non-isomorphisms $f_i\colon M_i\to M_{i+1}$. If for any element m in M_1 there exists n such that $f_nf_{n-1}\cdots f_1(m)=0$, we say $\{f_i\}_1^\infty$ is *locally T-nilpotent.* If for any countable sets $\{M_i\}_1^\infty$ above such that $\alpha(2i-1)\neq\alpha(2i'-1)$ ($\beta(2j)\neq\beta(2j')$) if $i\neq i'$ ($j\neq j'$) any sets $\{f_i\}$ of non-isomorphisms are always locally T-nilpotent, then we say $\{P_\alpha\}_I$ and $\{Q_\beta\}_J$ are *relatively and locally sami-T-nilpotent.* If we omit the assumptions $\alpha(2i-1)\neq\alpha(2i')$ ($\beta(2j)\neq\beta(2j')$) in the above, we say $\{P_\alpha\}_I$ and $\{Q_\beta\}_J$ are *relatively and locally T-nilpotent.* If $\{P_\alpha\}_I=\{Q_\beta\}_J$, we say $\{P_\alpha\}_I$ is *locally semi-T-nilpotent* or *T-nilpotent,* corresponding to the above cases. We shall assume that the definition of relatively semi-T-nilpotency contains a case of either I or J being finite. If $K=\sum_I\oplus P_\alpha=\sum_J\oplus Q_\beta$ and $\{P_\alpha\}_I$, $\{Q_\beta\}_J$ are locally and relatively T-nilpotent, then we say $\sum_I\oplus P_\alpha$ and $\sum_J\oplus Q_\beta$ are *relatively T-nilpotent decompositions* of K.

Finally, let $M=N\oplus P$ be R-modules and κ a cardinal number. If for any decomposition $M=\sum\oplus L_\alpha$ with κ-components there exist submodules L_α' of L_α such that $M=N\oplus\sum\oplus L_\alpha'$, then we say N has *the κ-exchange property* in M. In case κ is any cardinal, we say N has *the exchange property in M.*

1. T-nilpotent decompositions

First, we study a property of relative T-nilpotency. If the endomorphism ring of a module M is a local ring, then we call M *completely indecomposable.*

Lemma 1. *Let M be an R-module and f,g in* $\mathrm{End}_R(M)$. *If fg is isomorphic, $M=\mathrm{Im}\,g\oplus\mathrm{Ker}\,f$.*