

ON A GENERALIZATION OF QF-3' RINGS^{*)}

Dedicated to Professor Kiiti Morita for the celebration of his sixtieth birthday.

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A ring R with identity is called left QF-3' if the injective hull $E(R)$ of the left R -module R is torsionless. This class of rings and the other related generalizations of quasi-Frobenius rings have been studied by a number of authors.

Recently, Jans [7] has given a torsion theoretic characterization of left QF-3' rings (cf. also Kato [8] and Tsukerman [14]). The purpose of this paper is, generalizing this idea, to consider a module theoretic generalization of left QF-3' rings. We shall say that a left R -module Q is QF-3' if its injective hull $E(Q)$ is torsionless with respect to Q , i.e., $E(Q)$ can be embedded in a direct product of copies of Q .

The main theorem of §1 will give some equivalent conditions for Q to be QF-3'.

In §2, we shall discuss basic properties of QF-3' R -modules and study a relation between QF-3' R -modules and cogenerators for R -mod.

We shall treat, in §3, QF-3' R -modules with zero singular submodule. We shall give some results relating the notions of Q -torsionless R -modules and non-singular R -modules. In particular we shall show that, if Q is faithful, these notions coincide if and only if Q is QF-3' and has zero singular submodule. We shall also give another characterization of a QF-3' R -module with zero singular submodule making use of its injective submodules.

After completed this paper, we found that the similar results were obtained by Bican [2] and wrought a slight change in the paper.

Throughout this paper, R will denote an associative ring with identity and R -mod the category of unital left R -modules and R -homomorphisms. We shall deal only with left R -modules and so R -modules will mean unital left R -modules. $E(M)$ will always denote the injective hull of a left R -module M and $r_M(*)$ the right annihilator of $*$ in M .

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