Kanzaki, T.
Osaka J. Math.
12 (1975), 691–702

ON GALOIS EXTENSION WITH INVOLUTION OF RINGS

Dedicated to Professor Kiiti Morita on his 60th birthday

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(Received December 3, 1974)

1. Introduction

For a Galois extension field L of a field K with Galois gruop G, A. Rosenberg and R. Ware [9] proved that if $[L:K]$ is odd then the Witt ring $\mathrm{W}(K)$ is isomorphic to $\mathrm{W}(L)^G$. The proof was simplified by M. Knebusch and W. Scharlau [5], and the theorem was generalized by M. knebusch, A. Rosenberg and R. Ware [6] to the case of commutative semilocal rings. In this note, concerning with sesqui-linear forms over a non commutative ring defined in [2], we want to extend the theorem to a case of non commutative rings. In §2 and §3, we difine a *Galois extension with involution* of a ring and an *odd type Galois extension with involution.* From the theorem of Scharlau (cf. [11], [7]), we know that for a Galois extension with involution $L\supset K$ of fields, $L\supset K$ is an odd type Galois extension with involution if and olnly if $[L:K]$ is odd. If $A\supset B$ is a G-Galois extension with involution of rings, then we can prove the isomorphism $i^*\circ t_{G*}(q)=\sum_{\sigma\in G}\perp\sigma^*(q)$ for any sesqui-linear left A-module $q=(M, q)$. This isomorphism is a generalization of the case of fields [4], semilocal rings [6]. If A is an algebra over a commutative ring R, and if $A\supset R$ is an odd type G-Galois extension with involution, then it is obtained that the inclusion map i: $R\to A$ induces a group monomorphism i^*: $\mathrm{W}(R)\to\mathrm{W}(A)$ of Witt groups of hermitian left modules, and its image is $T_{G^*}(\mathrm{W}(A))$. Throughout this paper, we assume that every ring has identity element and module is unitary. Furthermore, ring homomorphisms are assumed to correspond identity element to identity element.

2. Sesqui-linear forms

DEFINITION 1. Let A be a ring with involution $A\to A$; $a\rightsquigarrow\bar{a}$, i.e. $\overline{a+b}=\bar{a}+\bar{b}$, $\overline{ab}=\bar{b}\,\bar{a}$ and $\bar{\bar{a}}=a$ for every a, b in A. For a subring B and a finite group G of ring-automorphisms of A, $A\supset B$ is called a *G-Galois extension with involution* if every element in G is compatible with the involution, i.e. $\overline{\sigma(a)}=\sigma(\bar{a})$ for all $a\in A$, $\sigma\in G$, and if $A\supset B$ a G-Galois extension, i.e. $A^G=B$ and there exist