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UNIVERSAL COEFFICIENT SEQUENCES FOR COHOMOLOGY THEORIES OF *CW*-SPECTRA

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Kainen [5] showed that there exists a cohomology theory $k^*($ (?) and a natural short exact sequence

$$0 \to \mathrm{Ext}\,(h_{*-1}(X),G) \to k^*(X;G) \to \mathrm{Hom}\,(h_*(X),G) \to 0$$

for any based CW-complex X if h_* is an (additive) homology theory and G is an abelian group. On the other hand, for an (additive) cohomology theory k^* such that k^*(point) has finite type Anderson [3] constructed a homology theory Dk_* and a natural exact sequence

$$0 \to \mathrm{Ext}\,(Dk_{*-1}(\,F),\ Z) \to k^*(\,F) \to \mathrm{Hom}\,(Dk_*(\,F),\ Z) \to 0$$

for any finite CW-complex whose extension to arbitrary CW-complexes is given in a form of a four term exact sequence. He then determined homology theories Dk_* in the special cases $k^*=H^*$, K^* and KO^*. Ordinary cohomology theory and complex K-theory are both self-dual and real K-theory is the dual of sympletic K-theory, i.e., $DH_*=H_*$, $DK_*=K_*$ and $DKSp_*=KO_*$. Moreover he asserted that D^2 is the identity, i.e., $D(Dk)_*=k_*$.

In this note we shall construct a CW-spectrum $\hat{E}(G)$ for every CW-spectrum E and abelian group G by Kainen's method involving an injective resolution of G, and state a relation between E and $\hat{E}(G)$ in a form of a universal coefficient sequence

$$0 \to \mathrm{Ext}\,(E_{*-1}(X),G) \to \hat{E}(G)^*(X) \to \mathrm{Hom}\,(E_*(X),G) \to 0$$

for any CW-spectrum X. And we shall study some properties of $\hat{E}(G)$. For example, under a certain finiteness assumption on $\pi_*(E)$ we show that $\hat{E}(R)$ *(R)* has the same homotopy type of ER where J? is a subring of the rationals Q (Theorem 2). The above universal coefficient sequence combined with Theorem 2 gives us a new criterion for $ER^*(X)$ being Hausdorff (Theorem 3). Also we shall discuss uniqueness of $\hat{E}(G)$ (Theorem 4). Furthermore, using Anderson's technique we investigate the homotopy type of $\hat{E}(G)$ in the special cases $E=H$, K and KSp (Theorem 5). Finally we note that $K^{2n}(K\wedge\cdots\wedge K)$