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UNIFORM ALGEBRA GENERATED BY $z_1, \cdots, z_n, f_1(z), \cdots, f_s(z)$

Dedicated to Professor Yukinari Tôki on his 60th birthday

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Introduction

Let $\boldsymbol{C}^n$ be the complex Euclidean space with complex coordinates $z=(z_1, \cdots, z_n)$ and K a compact subset of $\boldsymbol{C}^n$. For any complex valued C^∞-functions $f_1, \cdots, f_s$ defined on an open subset U of $\boldsymbol{C}^n$ containing K, we shall consider the uniform algebra A consisting of uniform limits of polynomials of $z_1, \cdots, z_n, f_1, \cdots, f_s$ on K.

Hörmander-Wermer [1] proved that, if $s=n$ and if each f_j is 'close' to $\bar{z}_j$ in some sense, then A coincides with $C(K)$, the algebra of all complex valued continuous functions on K. In this paper, we shall deal with the case where $0<s<n$ and each f_j is holomorphic in $z_{s+1}, \cdots, z_n$ near K. In Section 3, an approximation theorem on the graph of $f_1, \cdots, f_s$ will be proved. In Section 4, we shall give a sufficient condition on f_j and K assuring that every function holomorphic in $z_{s+1}, \cdots, z_n$ near K belongs to A.

1. The graph of $f_1, \cdots, f_s$

Let $f_1, \cdots, f_s$ be C^∞-functions defined on an open subset U of $\boldsymbol{C}^n$. The graph of $f_1, \cdots, f_s$

$$M = \{(z_1, \cdots, z_n, f_1(z), \cdots, f_s(z)) \in \boldsymbol{C}^{n+s};\ z = (z_1, \cdots, z_n) \in U\}$$

is a real $2n$-dimensional submanifold of $\boldsymbol{C}^{n+s}$. If g is a C^∞-function on M, then the function g_0 defined by

$$(1.1) \qquad g_0(z_1, \cdots, z_n) = g(z_1, \cdots, z_n, f_1(z_1, \cdots, z_n), \cdots, f_s(z_1, \cdots, z_n))$$

is a C^∞-function on U.

We denote by $H_r(U)$, $r=n-s$, the class of functions of $C^\infty(U)$ which are holomorphic in $z_{s+1}, \cdots, z_n$.

We shall now consider the following assumptions on $f_1, \cdots, f_s$:

$$(1.2) \qquad f_1, \cdots, f_s \text{ belong to } H_r(U), \text{ and}$$