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# A UNIQUE CONTINUATION THEOREM FOR AN ELLIPTIC OPERATOR OF TWO INDEPENDENT VARIABLES WITH NON-SMOOTH DOUBLE CHARACTERISTICS

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**1. Introduction.** Let $P=P(x; \partial/\partial x)$ be an elliptic homogeneous differential operator of order $m(\geqq 2)$ with complex valued $C^\infty$ coefficients defined near the origin in the 2-dimensional real Euclidean space $R^2$. We say that $P$ is A-ellilpic at $x_0$ if $x_0$ has a neighbourhood $U$ such that for any open and connected neighbourhood $V$ $(\subset U)$ of $x_0$, there is no non-trivial solution $u \in C^m(V)$ of the differential inequality in $V$

$$(1) \qquad |P(x; \partial/\partial x)u| \leqq C \sum_{|\alpha|<m} |(\partial/\partial x)^\alpha u|$$

such that $u=0$ in some open subset (of $V$) whose closure contains the point $x_0$. It is well known that $P$ is A-elliptic at each point where $P$ has simple characteristics, or $P$ has double characteristics and has Lipschitz continuous characteristic roots (see Hörmander [1], Pederson [2]).

In the present paper we shall give a sufficient condition for the operator $P$ to be A-elliptic when $P$ has double characteristics and its symbol $P(x; \xi)$ has a factorization of the form in a neighbourhood of the origin

$$(2) \quad p(x; \xi) = a(x) \prod_{j=1}^{N} (\xi_1^2 + 2a_j(x)\xi_1\xi_2 + b_j(x)\xi_2^2)$$

or

$$(3) \quad p(x; \xi) = a(x) \prod_{j=1}^{N} (\xi_1^2 + 2a_j(x)\xi_1\xi_2 + b_j(x)\xi_2^2) \prod_{j=N+1}^{N+s} (\xi_1 + a_j(x)\xi_2)\,.$$

Here $a$, $a_j$ and $b_k$ are $C^\infty(\omega)$ functions such that $a(0) \neq 0$, $a_j(0)^2 = b_j(0)$ $(j=1, \cdots, N)$ and $a_j(0) \neq a_k(0)$ $(1 \leq j \neq k \leq N+s)$.

Set $c_j(x) = b_j(x) - a_j(x)^2$ and let $R_j$ be the set of points $y \in \omega$ which has a neighbourhhood where $c_j(x) = k(x)^2$ for some $C^{1+1/2}$ function $k(x)$. Then we have our main result as follows.

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