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ON THE POTENTIAL TAKEN WITH RESPECT TO COMPLEX-VALUED KERNELS

MINORU MATSUDA

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In the potential theory, we have two theorems called the existence theorem concerning the potential taken with respect to real-valued and symmetric kernels. They are stated as follows. Let $K(X, Y)$ be a real-valued function defined in a locally compact Hausdorff space Ω, lower semi-continuous for any points X and Y, may be $+\infty$ for $X=Y$, always finite for $X\neq Y$ and bounded from above for X and Y belonging to disjoint compact sets of Ω respectively. For a given positive measure μ, the potential is defined by

$$K\mu(X) = \int K(X, Y)d\mu(Y),$$

and the K-energy of μ is defined by $\int K\mu(X)d\mu(X)$. A subset of Ω is said to be of positive K-transfinite diameter, when it charges a positive measure μ of finite K-energy with compact support, otherwise said to be of K-transfinite diameter zero. Let $K(X, Y)$ be symmetric : $K(X, Y)=K(Y, X)$ for any points X and Y. Then we have two following theorems.

Theorem A. *Let F be a compact subset of positive K-transfinite diameter, and $f(X)$ be a real-valued upper semi-continuous function with lower bound on F. Then, given any positive number a, there exist a positive measure μ supported by F and a real constant γ such that*
(1) $\mu(F)=a$,
(2) $K\mu(X)\geqq f(X)+\gamma$ *on F with a possible exception of a set of K-transfinite diameter zero, and*
(3) $K\mu(X)\leqq f(X)+\gamma$ *on the support of μ.*

Theorem B. *In the above theorem, suppose the further conditions : $K(X, Y)>0$ and $\inf f(X)>0$ for any points X and Y of F. Then, given any compact subset F of positive K-transfinite diameter, there exists a positive measure μ supported by F such that*
(1) $K\mu(X)\geqq f(X)$ *on F with a possible exception of a set of K-transfinite diameter zero, and*