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ON SOME EXTREMAL QUASICONFORMAL MAPPINGS OF DISC

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1. Introduction

A quasiconformal mapping $w(z)$ of the unit disc $\Delta=\{z \mid |z|<1\}$ onto itself is known to have continuous boundary values, hence we may consider the class $Q(w; \Delta, \Delta)$ of all quasiconformal mappings of Δ onto itself that coincide with $w(z)$ on the boundary $\partial\Delta=\{z \mid |z|=1\}$. In $Q(w; \Delta, \Delta)$ there is at least one quasiconformal mapping whose maximal dilatation is a minimum. Such a quasiconformal mapping is called extremal in the class $Q(w; \Delta, \Delta)$. If there exists a regular single-valued analytic function φ defined on Δ and if the complex dilatation μ of a quasiconformal mapping is written in the form

$$\mu = k\frac{\bar{\varphi}}{|\varphi|} \quad (0<k<1)\,, \tag{1}$$

except at zeros of φ, then it is called a Teichmüller mapping corresponding to φ. It was studied by K. Strebel [4] whether a quasiconformal mapping $f(z)$ with the complex dilatation of the form (1) is extremal in the class $Q(f; \Delta, \Delta)$ or not.

In section 2 and section 3 we prove two distortion theorems which serve to show some extremality. In section 4 some extremal quasiconformal mappings which are not Teichmüller mappings in general are considered.

2. Distortion of argument (1)

Let $w(z)$ be a K-quasiconformal mapping which maps $|z|<1$ onto $|w|<1$ with $w(0)=0$ and $w(1)=1$ and let $\arg w(z)=\arg w(re^{i\theta})$ a continuous branch with $\arg w(1)=0$. Then we have

Theorem 1. *For all K-quasiconformal mappings which map $|z|<1$ onto $|w|<1$ with $w(0)=0$ and $w(1)=1$, we have*

$$\varlimsup_{r\to 0}\left|\frac{\arg w(r)}{\log r}\right| \leqq \frac{1}{2}\left(K-\frac{1}{K}\right). \tag{2}$$