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EXTENDIBLE VECTOR BUNDLES OVER LENS SPACES MOD 3

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1. Introduction. In [5] Schwarzenberger investigated the problem of determing whether a real vector bundle over the real projective space RP^n can be extended to a real vector bundle over RP^m $(n<m)$. In [3], he also investigated the case of the complex tangent bundle of the complex projective space.

The purpose of this note is to prove the non-extendibility of a bundle over lens spases mod 3 by making use of Schwarzenberger's technique ([5]).

Let S^{2n+1} be the unit $(2n+1)$-sphere. That is

$$S^{2n+1}=\{(z_0, \cdots, z_n);\ \sum_{i=0}^{n}|z_i|^2=1,\ z_i\in C \quad \text{for all } i\}$$

Let γ be the rotation of S^{2n+1} defined by

$$\gamma(z_0, \cdots, z_n)=(e^{2\pi i/p}z_0 \cdots, e^{2\pi i/p}z_n)\,.$$

Then γ generates the differentiable transformation group Γ of S^{2n+1} of order p, and lens space mod p is defined to be the orbit space $L^n(p)=S^{2n+1}/\Gamma$ It is a compact differentiable $(2n+1)$-manifold without boundary and $L^n(2)=RP^{2n+1}$. The Grothendieck rings $\widetilde{KO}(L^n(p))$, $\tilde{K}(L^n(p))$ were determined by T. Kambe [4]. We recall them in **2**. Let $\{z_0, \cdots, z_n\}\in L^n(p)$ denote the equivalence class of $(z_0, \cdots, z_n)\in S^{2n+1}$. $L^n(p)$ is naturally embedded in $L^{n+1}(p)$ by identifying $\{z_0, \cdots, z_n\}$ with $\{z_0, \cdots, z_n, 0\}$. Hence $L^n(p)$ is embedded in $L^m(p)$ for $n<m$. Throughout this note we suppose $p=3$. Now we state our theorems which shall be proved in **3** and **4**.

Let ζ be any t-dimensional real bundle over $L^n(3)$. Let $p(\zeta)$ be the mod 3 Pontryagin class of ζ

$$p(\zeta)=\sum_j p_j(\zeta) \text{ where } p_j(\zeta)=(-1)^j C_{2j}(\zeta\otimes C) \quad \bmod 3\,.$$

From the property of the cohomology algebra $H^*(L^n(3)\colon Z_3)$, we have

$$p_j(\zeta)=d_j x^{2j}\,,$$