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ON THE GALOIS THEORY OF NON-COMMUTATIVE RINGS

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In this paper, some results of Galois theory for commutative rings without idempotents developed by Chase, Harrison and Rosenberg in [1], are generalized to non-commutative rings that verify a certain condition. Some proofs are similar to those appearing in [1].

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1. Introduction

In this section, the previous definitions, already known, are remembered. As usual, all rings have units, all modules are unitary and ring homomorphisms carry the unit into the unit.

Let S be a ring, G a finite group of automorphisms of S and $R=S^G$, the fixed subring. We say that S is a Galois extension of R with group G, if there exist elements x_i, y_i $(i=1, 2, \cdots, n)$ in S, such that:

$$\sum_i x_i\sigma(y_i) = \delta_{1,\sigma}, \qquad \text{for all } \sigma\in G .$$

We indicate with $D=D(S, G)$ the crossed product of S with basis $(u_\sigma)_{\sigma\in G}$ and with tr the trace map, that is to say, the map of S into R defined by $tr(x) =\sum_{\sigma\in G} \sigma(x)$.

$S^\cdot$ shall denote the S structure as a right module, on the ring mentioned in each case.

The application $d{:}D\rightarrow \mathrm{Hom}_R(S^\cdot, S^\cdot)$, defined by $d(s\cdot u_\sigma)(x)=s\sigma(x)$, for each s, x in S and each σ in G, is a ring homomorphism and two-sided S-homomorphism.

As in [1], E designates the set of all functions of G into S. Then E is a ring