Kôno, N.
Osaka J. Math.
6 (1969), 1-12

OSCILLATION OF SAMPLE FUNCTIONS IN STATIONARY GAUSSIAN PROCESSES

NORIO KÔNO

(Received August 29, 1968)

1. Introduction

There are many sample function properties of a stationary Gaussian process which satisfy 0–1 law. For example, continuity or unboundedness of sample functions, upper class or lower class and the law of iterated logarithm. In this paper we shall investigate another type of sample property which satisfies 0–1 law.

Let $\{X(t);\ 0\leq t\leq 1\}$ be a real stationary Gaussian process with the mean $E[X(t)]=0$ which has continuous sample functions with probability one, and let $Q(x)$, $0\leq x<+\infty$, be a continuous increasing function near the origin with $Q(0)=0$. We shall investigate the oscillation of sample functions of $X(t)$ described as follows;

$$\lim_{||S_n||\to 0}\sum_{i=1}^{m(n)} Q(|X(t_i^{(n)})-X(t_{i-1}^{(n)})|), \tag{1}$$

where $S_n=\{0=t_0^{(n)}<t_1^{(n)}<\cdots<t_{m(n)}^{(n)}=1\}$ is a partition of $[0, 1]$ and $||S_n||=\operatorname*{Max}_{i=1,\dots,m(n)}|t_i^{(n)}-t_{i-1}^{(n)}|$.

In Theorem 1 we shall prove that if $Q(x)$ is suitably chosen, the oscillation (1) satisfies Kolmogorov's 0–1 law for a certain class of stationary Gaussian processes. This class is specified by the conditions on $v(x)=(E[(X(x)-X(0))^2])^{1/2}$ using a regular increasing function. In Theorem 2 we shall prove that the oscillation (1) has non-zero finite constant with probability one under the stronger conditions of $v(x)$ than those of Theorem 1 and with a nice choice of the seguence of partitions.

In the case that $\{X(t);\ 0\leq t\leq 1\}$ is the Wiener process, the oscillation (1) for $Q(x)=x^2$ equals 1 with probability one when $\{S_n\}$ is the 2^n equi-partitions (P. Lévy [1]). G. Baxter [2] showed that for the comparatively narrow class of not necessarily stationary Gaussian processes characterized by the conditions on $r(s, t)=E[X(s)X(t)]$, the oscillation (1) for $Q(x)=x^2$ is constant with probability one. E.G. Gladyshev [3] has extended the result of G. Baxter in the direction of the oscillation for $Q(x)=a(n)x^2$, where $a(n)$ is a normalized constant which