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A CONSTRUCTION OF REFLECTING BARRIER BROWNIAN MOTIONS FOR BOUNDED DOMAINS

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1. Introduction

Let D be an arbitrary bounded domain of the N-dimensional Euclidean space R^N.

We will call a function $G_\alpha(x, y)$ ($\alpha>0$, x, $y\in D$, $x\neq y$) a (continuous) *resolvent density* on D if the following conditions are satisfied:

(G. 1) $\quad G_\alpha(x, y)\geqq 0, \quad \alpha>0, \quad x, y\in D, \quad x\neq y\,.$

(G. 2) $\quad \alpha\int_D G_\alpha(x, y)\,dy\leqq 1, \quad \alpha>0, \quad x\in D\,.$[1]

(G. 3) $\quad G_\alpha(x, y)-G_\beta(x, y)+(\alpha-\beta)\int_D G_\alpha(x, z)\,G_\beta(z, y)\,dz=0\,,$

$\qquad \alpha, \quad \beta>0, \quad x, \quad y\in D, \quad x\neq y\,.$

(G. 4) For fixed $\alpha>0$, $G_\alpha(x, y)$ is continuous in (x, y) on $D\times D$ off the diagonal.

A resolvent density on D is called *conservative* if the equality holds in (G.2) for all $\alpha>0$ and all $x\in D$.

In this paper, we will construct a conservative resolvent density on D and show that it determines a diffusion process (that is, a strong Markov process having continuous trajectories) which takes values in a natural enlarged state space D^*. When the relative boundary ∂D of D is sufficiently smooth, our diffusion process is shown (Theorem 6) to be the well known reflecting barrier Brownian motion on $D\cup\partial D$. For this reason, our process for an arbitrary D may be considered the reflecting barrier Brownian motion in an extended sense.

A function $p(t, x, y)$, $t>0$, x, $y\in D$, will be called a (continuous) *transition density* on D, if it satisfies the following conditions:

(T. 1) $\quad p(t, x, y)\geqq 0, \ t>0, \ x, y\in D\,.$

1) dy denotes the Lebesgue measure on D.