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THE CHARACTERIZATION OF DIFFERENTIAL OPERATORS WITH RESPECT TO THE CHARACTERISTIC CAUCHY PROBLEM

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1. Introduction. Let $L(\lambda, \eta)=\sum_{j=0}^{M}\sum_{k=0}^{N} a_{j,k}\lambda^j\eta^k$ be a polynomial of λ and η with degrees M and N respectively. Then we can define a constant $\alpha(L)$ as follows. When $L(\lambda, 0)\not\equiv 0$, we set

$$\alpha(L) = \max_{a_{j,k}\neq 0, k>0} \frac{m-j}{k},$$

where m is the degree of $L(\lambda, 0)$. In this case we have $j+k\alpha(L)\leqq m$ if $a_{j,k}\neq 0$ and $j_0=k_0\alpha(L)=m$ for some (j_0, k_0) such that $k_0>0$ and $a_{j_0,k_0}\neq 0$. When $L(\lambda, 0)\equiv 0$, we define $\alpha(L)=-\infty$. It is easily shown by the definition of $\alpha(L)$ that the line $t=0$ is characteristic with respect to the differential operator $L\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right)$ if and only if $\alpha(L)<1$. L. Hörmander [3] proved that there exist null solutions[1] of the differential equation $Lu=0$ with respect to the half plane $\Pi=\{(t, x)\,;\, t\leqq 0\}$ if and only if the line $t=0$ is characteristic.

In this note we shall characterize the differential operator L by the smallest (largest) function class $G_x(\cdot)$[2] of Gevrey's to which null solutions are (not) able to belong. In theorem 1, using the same method as L. Hörmander's in [2], we construct a null solution which belongs to $G_x(\alpha+\varepsilon)$ for any $\varepsilon>0$ if $0<\alpha$[3] <1, and to $G_x(\alpha)$ if $-\infty\leqq\alpha\leqq 0$. In theorem 2, we prove the uniqueness of the solution of the Cauchy

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1) A solution $u(t, x)$ of the equation $Lu=0$ is called a null solution with respect to the half plane Π, if $u\in C^\infty(R^2)$ and $u\not\equiv 0$ in R^2 but $u=0$ in Π.

2) A C^∞-function $f(t, x)$ is called to be in $G_x(\alpha)$ in $(T_1, T_2)\times(x_1, x_2)$, $-\infty\leqq x_1<x_2\leqq +\infty$, if it satisfies

$$\left|\frac{\partial^k}{\partial x^k} f(t, x)\right| \leqq K^{k+1}(k!)^\alpha \qquad (k=0, 1, 2, \cdots)$$

in any finite interval $[a, b]$ in (x_1, x_2) for some constant K.

3) In what follows we write $\alpha=\alpha(L)$.