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K_U-GROUPS OF DOLD MANIFOLDS

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Introduction. J. F. Adams [1] calculated the Grothendieck rings K_U of the projective spaces. The manifold $D(m, n)$, defined by A. Dold in his study of cobordism theory [6], is regarded as a generalization of the projective spaces.

The purpose of this paper is to calculate K_U of the Dold manifold $D(m, n)$; the result is stated in Theorem (3.14) of §3. For this purpose, we construct a real 2-plane bundle η_1 over $D(m, n)$ which is a generalization of the real restriction of the canonical complex line bundle over $CP(n)$ and also of the bundle sum of the canonical real line bundle over $RP(m)$ and the trivial line bundle over $RP(m)$. This bundle η_1 plays an important role in computations. On the way of computations, we make use of mod 2 K_U-theory which is introduced by S. Araki and H. Toda [2].

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1. Cohomology rings of Dold manifolds

Let S^m, $m \geqq 0$, denote the unit m-sphere in R^{m+1} with the coordinates $x_0, x_1, \cdots, x_m$, and let $CP(n)$, $n \geqq 0$, denote the complex projective n-space with the homogeneous coordinates $z_0, z_1, \cdots, z_n$. Consider the product space $S^m \times CP(n)$ and difine a homeomorphism $T: S^m \times CP(n) \to S^m \times CP(n)$ by

$$T(x, z) = (-x, \bar{z}) \qquad (x \in S^m, z \in CP(n)), \tag{1.1}$$

where $-x$ is the antipodal point of x and $\bar{z}$ is the conjugate point of z. Then, by definition, the Dold manifold $D(m, n)$ is the quotient space obtained from $S^m \times CP(n)$ by identifying (x, z) with $T(x, z)$.

The projection $S^m \times CP(n) \to S^m$ induces naturally a map p of $D(m, n)$ onto the real projective m-space $RP(m)$, and $\{D(m, n), p, RP(m), CP(n),$