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NOTE ON UNIQUENESS OF SOLUTIONS OF DIFFERENTIAL INEQUALITIES OF PARABOLIC TYPE

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Recently A. Friedman [3] proved some result on the uniqueness of solutions of ordinary differential inequalities

$$\left\| \frac{1}{i}\frac{du}{dt} - A(t)u \right\| \leqq \eta \| A(t)u \| + K\|u\| \tag{0. 1}$$

in a Hilbert space, and as its application he showed that a certain uniqueness theorem holds for differential inequalities of parabolic type

$$\left\| \frac{\partial u}{\partial t} - P(x, t, D_x)u \right\|_0 \leqq \eta \| u(t) \|_{2m} + K\|u(t)\|_{2m-1} \tag{0. 2}$$

in the class of functions satisfying some type of time-independent boundary conditions

$$B_j(x, D_x)u(x, t) = 0, \quad x \in \partial\Omega, \quad j = 1, \cdots, m, \tag{0. 3}$$

where $\partial\Omega$ is the boundary of a bounded domain Ω and $\| \quad \|_k$ stands for the usual norm of $H_k(\Omega) = W_2^k(\Omega)$. In [3] $A(t)$ is assumed to have a constant domain, and on this account it was required that the boundary conditions (0. 3) did not depend on t. In this paper it will not be attempted to extend the above result concerning the inequalitites (0. 1) in a Hilbert space to the case in which $A(t)$ has a variable domain; however, it will be shown that a similar result remains valid for more general differential inequalities of parabolic type with time-dependent boundary conditions

$$\|A(x, t, D_x, D_t)u\|_0 \leqq \eta \sum_{k=0}^{l} \|D_t^{l-k}u\|_{kd} + K\sum_{k=0}^{l-1} \|D_t^{l-k-1}u\|_{kd}\,, \tag{0. 4}$$

$$B_j(x, t, D_x, D_t)u(x, t) = 0, \quad x \in \partial\Omega, \quad j = 1, \cdots, m. \tag{0. 5}$$

Here $A(x, t, D_x, D_t) = \sum_{k=0}^{l} A_{l-k}(x, t, D_x)D_t^k$ is a linear $d(=2m/l)$-parabolic differential operator in the sense of I. G. Petrowskii, or, what amounts to the same thing, for each t and θ with $-\pi \leqq \theta \leqq 0$ $A(x, t, D_x, e^{i\theta}D_y^d)$