DeMeyer, F. R.
Osaka J. Math.
2 (1965), 117-127

SOME NOTES ON THE GENERAL GALOIS THEORY OF RINGS

F. R. DEMEYER

(Received January 25, 1965)

Introduction

In [2] M. Auslander and O. Goldman introduced the notion of a Galois extension of commutative rings. Further work by D. K. Harrison [9] indicates that the notion of a Galois extension will have significant applications in the general theory of rings. T. Kanzaki, in this journal, proved a "Fundamental Theorem of Galois Theory" for an outer Galois extension of a central separable algebra over a commutative ring. We generalize, complete, and give a new shorter proof of this result. The inspiration for the improvements in Kanzaki's result came from a paper by S. U. Chase, D. K. Harrison and A. Rosenberg [4].

This author in [6] began the study of 'Galois algebras'. These are not necessarily commutative Galois (in the sense of [2]) extensions of a commutative ring. Here we continue that study by extending some of the results in [4] and by proving a generalized normal basis type theorem in this setting. This paper forms a portion of the author's Doctoral Dissertation at the University of Oregon. The author is indebted to D. K. Harrison for his advice and encouragement.

Section 0

Throughout Λ will denote a K algebra, C will denote the center of Λ ($C=\mathfrak{Z}(\Lambda)$). G will denote a finite group represented as ring automorphisms of Λ and Γ the subring of all elements of Λ left invariant by all the automorphisms in G ($\Gamma=\Lambda^G$).

Let $\Delta(\Lambda:G)$ be the crossed product of Λ and G with trivial factor set. That is

$$\Delta(\Lambda:G)=\Sigma_{\sigma\in G}\Lambda U_\sigma \qquad \text{such that}$$
$$x_1U_\sigma x_2U_\tau = x_1\sigma(x_2)U_{\sigma\tau} \qquad x_1,\ x_2\ \Lambda\ ;\ \sigma,\ \tau\in G\,.$$

This work was done while the author held a National Science Foundation Cooperative Fellowship.