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Osaka J. Math.
2 (1965), 71–115

MULTIPLICATIVE STRUCTURES IN MOD q COHOMOLOGY THEORIES I.

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(Received March 24, 1965)

Introduction. In the course of developments of algebraic topology, one of the advantages to use the cohomology theory (in the ordinary sense) rather than the homology theory was the existence of multiplicative structures, i.e., cup products. Cup products exist not only in the integral cohomology but also in the cohomology with any coefficients whenever the coefficient domain has a multiplicative structure. Recently it is known that for any general cohomology theory one can associate cohomology theories with coefficients. Since there are important general cohomology theories with multiplicative structures such as K-theories, one can expect to introduce multiplicative structures in their associated cohomology theories with coefficients. The present work is directed to introduce and to study multiplicative structures in cohomology theories with coefficients. However, since coefficients are limited to finitely generated groups at the present time, the most important cases are those with coefficients in some finite cyclic groups, i.e., Z_q, $q>1$. Henceforth our research is limited to mod q cohomology theories to avoid complexity of discussions.

To introduce multiplications in mod q cohomology theories it is important to check a connection with the multiplication in the original cohomology theory. Since there is the notion of "reduction mod q" also in general cohomology theories, we postulate this connection as the compatibility through reduction mod q, postulation (Λ_1). In the ordinary mod q cohomology theory, the mod q Bockstein homomorphism works as a derivation. Since this property has been proved to be much useful, we postulate a corresponding property also in general mod q cohomology theories, postulation (Λ_2). Proof of associativity of multiplications in mod q cohomology theories are very round about even if it is possible. But, to get some uniqueness type theorems, it is sufficient to postulate a weaker form of associativity, which we call "quasi-associativity," postulation (Λ_3). We have an example of mod q