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ONE FLAT 3-MANIFOLDS IN 5-SPACE

Dedicated to Professor Hidetaka Terasaka on his sixtieth birthday

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1. Introduction

The results concerned with closed orientable surfaces in 4-space obtained in [5] will be extended in the paper.

Things will be considered only from the piecewise-linear (or semi-linear) and combinatorial point of view, and manifolds M, W etc., will be combinatorial, orientable with an orientation, maps will be piecewise-linear with respect to (simplicial) subdivisions and generally homeomorphisms between manifolds will be orientation preserving. So that $M \subset M_1$, $M = M_2$ and $\partial M = M_3$ will indicate obvious relations between the orientations of manifolds, if meaningful, together with the usual set theoretic meanings, where ∂M is the boundary of M.

Let M_i be a closed n-manifold in an $(n+2)$-manifold W_i without boundary, $i=1, 2$. Precisely, there are subdivisions K_i and L_i of M_i and W_i respectively such that K_i is a subcomplex of L_i. For convenience, the situation is simply said that $M_i = |K_i|$ is in $W_i = |L_i|$ in the rest of the paper. Then M_1 is *iso-neighboring* to M_2 if there are regular neighborhoods U_i of M_i in W_i, see [4], where $U_i \subset W_i$ and an onto homeomorphism $\psi: U_1 \to U_2$ such that $\psi(M_1) = M_2$. By Theorem 1 of [4], the iso-neighboring relation is an equivalence relation.

In § 2 two invariances the collection of singularities and the Stiefel-Whitney class under the iso-neighboring relation will be dealt with. Let a closed n-manifold $M = |K|$ be in an $(n+2)$-manifold $W = |L|$ without boundary. For each point x of M, the links $Lk(x, K)$ $Lk(x, L)$ in K, L are $(n-1)$-, $(n+1)$-spheres respectively. Then M is said to be *p-flat* in W if the link $Lk(x, K)$ bounds an n-cell in $Lk(x, L)$, alternatively the $(n-1, n+1)$-knot $(Lk(x, K), Lk(x, L))$ is trivial, where $x \in M - |K^{p-1}|$ and K^q is the q-skeleton of K (K^{-1} is the empty set). The p-flatness of M in W is clearly invariant under the iso-neighboring relation. A 0-flat M in W is alternatively said to be *locally flat*. For a 1-flat M in W the *collection of singularities* of M in W will be defined, which is an in-