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THE BIVARIATE ORTHOGONAL INVERSE EXPANSION AND THE MOMENTS OF ORDER STATISTICS

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1. Introduction

The orthogonal inverse expansion has been introduced in my previous paper [9] to obtain the universal upper bounds and the approximation for the moments of order statistics. For the same purpose two other series have been introduced by David and Johnson [2] and Plackett [6] and the error of the approximation of $E(X_{r/n})$ is evaluated by Saw [8] and Plackett [6].

In this paper we shall derive the universal upper bounds and the approximation for $E(X^i_{r/n}X^j_{s/n})$ $(i, j=1, 2)$ together with the error of the approximation by means of the bivariate orthogonal inverse expansion.

2. Some preliminaries

First we restate for convenience the following Proposition in [9].

Proposition 1. *Let H be a pre-Hilbert space and $\{\varphi_\nu\}_{\nu=0,1,\cdots}$ be any orthonormal system in H. Put $a_\nu=(f, \varphi_\nu)$ and $b_\nu=(g, \varphi_\nu)$ for any elements f, g in H. Then we have*

$$(2.1)\qquad |(f, g)-\sum_{\nu=0}^{k} a_\nu b_\nu| \leqq \{\|f\|^2-\sum_{\nu=0}^{k} a_\nu^2\}^{1/2}\{\|g\|^2-\sum_{\nu=0}^{k} b_\nu^2\}^{1/2},$$

where equality holds if and only if $f, g, \varphi_0, \cdots, \varphi_k$ are linearly dependent.

We also use the following well-known Proposition concerning a bivariate orthonormal system in a rectangular domain. The proof is found, for example, in Courant and Hilbert [1].

Proposition 2. *Let $L^2(0, 1)$ and $L^2(R)$ be the Hilbert spaces of all functions square integrable in the interval $(0, 1)$ and the square $R=\{(u, v)\,|\,0\leqq u, v\leqq 1\}$, respectively. If $\{\varphi_\nu(u)\}_{\nu=0,1,\cdots}$ is a complete orthonormal system in $L^2(0, 1)$, then $\{\varphi_\lambda(u)\varphi_\nu(v)\}_{\lambda,\nu=0,1,\cdots}$ is a complete orthonormal system in $L^2(R)$.*