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UNSTABLE HOMOTOPY GROUPS OF UNITARY GROUPS
(odd primary components)

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1. Introduction

The purpose of this paper is to prove the following

Theorem. *For each odd prime p,*

$$^{p}\pi_{2n+2k-3}(U(n)) = Z_{p^N}$$

for $k \leqq p(p-1)$, $n > k$ *and* $n+k \equiv 0 \mod p$, *where* $N = \min\left(\left[\frac{k-1}{p-1}\right], \nu_p(n+k)\right)$ *and* $\nu_p(x)$ *is the highest exponent of p dividing the integer x.*

This theorem contains one of the result of [5] as a special case. We shall use the following well-known isomorphism.

$$\begin{aligned}\pi_{2n+2k-3}(U(n)) &\approx \pi_{2n+2k-2}(EP_{n+k}/EP_n) \quad \text{for} \quad n \geqq k-2 \ [8] \\ &\approx \pi_{2n+2k-2}(E(P_{n+k,k})) \\ &\approx \pi_{2n+2k-3}(P_{n+k,\,k}) \quad \text{for} \quad n > k \ [4],\end{aligned}$$

where E is the suspension, P_m $(m-1)$ complex dimensional projective space, EP_{n+k}/EP_n or $P_{n+k,\,k}$ the space obtained from EP_{n+k} or P_{n+k} by smashing the subcomplex EP_n or P_n to a point.

In §2 we recall some material from the homotopy theory of the sphere and the K-theory, and deduce some results which are used in §3. In §3 we prove the Theorem.

2. Preliminary material

2.1. Denote by $\alpha_{n+k,\,r}$ the coefficient of x^{n+k-1} in $(e^x-1)^{n+k-r}$ for $1 \leqq r \leqq t$. For any non zero rational number x, if $x = p^r \cdot q^s \cdots$ is the factorization of x into prime powers, we define $\nu_p(x) = r$. By (5.3), (5.4), (6.4) and (6.5) in [1], if $\nu_p(\alpha_{n+k,\,r}) \geqq 0$ for $1 \leqq r \leqq t$ and a fixed prime p, then we have that $\nu_p(\alpha_{n+k,\,t+1}) \geqq 0$ with the exceptional case $t = s(p-1)$,