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Osaka Math. J.
11 (1959), 63-69.

On Integral Basis of Algebraic Function Fields with Several Variables

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Let K be an algebraic function field with two variables and w a discrete valuation of rank 2 of K. Let L be a finite extension of K and $w_1, w_2, \cdots, w_g$ all the extensions of w in L. We denote the valuation rings of w in K and of w_i in L by $\mathfrak{o}_0$ and $\mathfrak{o}_i$. It is clear that $\mathfrak{o}=\bigcap_{i=1}^{g}\mathfrak{o}_i$ is the integral closure of $\mathfrak{o}_0$ in L. The structure of $\mathfrak{o}$ as an $\mathfrak{o}_0$-module will be determined in this paper. Let $(e_1^{(i)}, e_2^{(i)})$ be the value of ramification of w_i: $w_i(a)=(e_1^{(i)}, e_2^{(i)})w(a)$ for $a \in K$.[1] The main theorem given in this paper is that $\mathfrak{o}$ is a direct sum of $n_0(=\sum_i e_1^{(i)}f_i)$ $\mathfrak{o}_0$-modules of rank 1 and of $n-n_0$ $\mathfrak{o}_0$-modules of infinite rank where $n=[L:K]$. From this we can easily conclude that L/K has integral basis with respect to w in the classical sense when and only when $e_2^{(i)}=1$ for every i.[2] In order to get the theorem, some lemmas on the independence of valuations of rank 2 will be required, which are proved generalizing naturally the well-known proofs in case of rank 1. Then we construct concretely n linearly independent basis of L/K which are a generalization of the classical integral basis, having the following property: If $u_1, u_2, \cdots, u_n$ are those generalized integral basis and $\sum c_i u_i \in \mathfrak{o}$ with c_i in K, then $c_i u_i \in \mathfrak{o}$ for each i.

The above mentioned results will be inductively generalized in general case. Let K be an algebraic function field with several variables and w a discrete valuation of K. Let L be a finite extension of K. We must assume that there holds a fundamental equality with respect to the extensions of w in L: $\sum_i e_i f_i = n$ where e_i are the ramification indices of w_i and f_i are the relative degrees of w_i. This equality holds when rank $w + \dim w = n$. (See Roquette [4]. p. 43. Second Criterion.) In this case the main theorem is proved to be true, although we do not discuss the general case in this paper.

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1) We express $w_i(a)$ and $w(a)$ in the normal exponential form and the order of value group will be determined by the last non-zero difference of components (contrary to the usual sense. [5]).

2) For the definition of integral basis, see [1].