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*An Investigation on the Logical Structure of Mathematics (IX)**

Deductions in the Natural-Number Theory $T_1(N)$

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The consistency of the natural-number theory $\mathrm{T}_1(\mathrm{N})$ is proved in Section B, Part (VIII), and the natural-number theory $T_1(\mathrm{N})$ in a generalized sense is defined in §2, Part (VIII), where $T_1(\mathrm{N})$ denotes the natural-number-theoretic extension of any arbitrary elementary natural-number theory, so that the consistency of $T_1(\mathrm{N})$ can be proved by the same method as that of $\mathrm{T}_1(\mathrm{N})$. Thus, if we have a series of elementary natural-number theories $\mathrm{T}_0(\mathrm{N})\subset\mathrm{T}_0'(\mathrm{N})\subset\mathrm{T}_0''(\mathrm{N})\subset\cdots$, then we have the series of consistent natural-number theories $\mathrm{T}_1(\mathrm{N})\subset\mathrm{T}_1'(\mathrm{N})\subset\mathrm{T}_1''(\mathrm{N})\subset\cdots$. By $T_1(\mathrm{N})$ we denote any one of these natural-number theories $\mathrm{T}_1(\mathrm{N})$, $\mathrm{T}_1'(\mathrm{N})$, $\mathrm{T}_1''(\mathrm{N})$, $\cdots$. But $T_1(\mathrm{N})$ does not denote representatively a subsystem of UL which belongs to a *formally* defined class of subsystems of UL; the notation $T_1(\mathrm{N})$ is a word belonging to the intuitive language. After fixing some natural-number theory $\mathrm{T}_1^{(i)}(\mathrm{N})$ within $T_1(\mathrm{N})$, the possibility of extending further this $\mathrm{T}_1^{(i)}(\mathrm{N})$ within $T_1(\mathrm{N})$ remains always open. A fixed $\mathrm{T}_1^{(i)}(\mathrm{N})$ has generally various directions of extension within $T_1(\mathrm{N})$, while some extension of $\mathrm{T}_1^{(i)}(\mathrm{N})$ may not remain within $T_1(\mathrm{N})$ (Beendigung) or, more strongly, may become inconsistent[1] (Hemmung).

This Part is divided into two Sections A and B. In Section A we treat addition and in Section B multiplication. In Section A addition is discussed in detail, while in Section B multiplication is discussed briefly to such an extent that we can know that the multiplication can be treated quite in a similar method as in Section A.

The formulas in Section A are numbered as $\mathrm{N}+k$ and those in Section B as $\mathrm{N}\times k$, where k is the number of a formula in each Section.

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* Continuation of Part (VIII), Nagoya Math. J. **14** (1959), 129–158. Other Parts referred to in this Part are as follows: Part (II), Hamburger Abh. forthcoming; Parts (III) and (IV), Nagoya Math. J. **13** (1958); Part (VII), ibid. **14** (1959).

1) Analogy is found in the definition of Brouwer's *spread* (*Menge*). See, for instance, A. Heyting: Intuitionism, Studies in Logic and Foundations of Mathematics, Amsterdam (1956), pp. 32–37, or L. E. J. Brouwer: Zur Begründung der intuitionistischen Mathematik I. Math. Ann. **93** (1925), pp. 244-5.