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Strict Convexity and Smoothness of Normed Spaces

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V. L. Klee [11][1] and M. M. Day [6] have considered various problems on strict convexity and smoothness of normed spaces. In his paper, Day [6] raised several questions. Two of these are the following:

(1) Is any L_1-space strictly convexifiable?

(2) Is there a nonreflexive nonsparable *scm* space?

In this paper, we consider these questions. In §2 we deal with spaces of bounded continuous functions and consider strict convexity and smoothness on these spaces. In §3 we give a partial answer to the first question, and in §4 we give an answer to the second, by showing an example of a nonreflexive nonseparable *scm* space.

§1. Preliminary.

Let E be a normed space. If every chord of the unit sphere has its midpoint below the surface of the unit sphere, then E is called *strictly convex* (written *SC*); if through every point of the surface of the unit sphere of E there passes a unique hyperplane of support of the unit sphere, then E is called *smooth* (written *SM*); if both occur, then E is called *SCM*. If E is isomophic to an *SM*, an *SC* and an *SCM* space, then E is called an *sm*, an *sc* and an *scm* space respectively.

If I is an index set, we define:

$m(I)=$ the space of all bounded real functions on I with $\|x\|=\text{l.u.b.}_{i\in I} |x(i)|$.

$c_0(I)=$ the subspace of those x in $m(I)$ for which for each $\varepsilon>0$ the set of i with $|x(i)|>\varepsilon$ is finite; that is, $c_0(I)$ is the set of functions vanishing at infinity on the discrete space I.

$l_p(I)$ (for $p\geqq 1$) $=$ the set of those real functions x on I for which $\|x\|_{l_p}=[\sum|x(i)|^p]^{1/p}<+\infty$.

Let X be a topological space. Then $C(X)$ denotes the space of all real-valued bounded continuous functions on X such that the norm $\|f\|=\sup_{x\in X}|f(x)|$.

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1) Numbers in bracket refer to the references cited at the end of the paper.