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On Differentiation of Set-Functions with some of its Applications

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If we want to generalize the known theorems on the relation between differentiation and integration to the case where the measure is defined on a completely additive class of sets in an abstract space without topology, we ought to avoid the use of the covering theorem of Vitali's type, which seems to depend more or less upon the dimensional consideration of the space.

In 1936 RENE DE POSSEL in his elegant paper: *Sur la dérivation abstraite des fonctions d'ensemble* (Journ. d. Math.) obtained some general results which fulfil such demands. The aim of the present paper is to generalize and develop his theory further and give some of its applications.

§ 1. Preliminary

We shall mean by $\mathfrak{F}$ a completely additive class of sets contained in a fixed set E and satisfying the following two conditions: i) $A, B \in \mathfrak{F}$ implies $A-B \in \mathfrak{F}$, ii) $A_j \in \mathfrak{F}$ $(j=1, 2, \dots)$ implies $\bigcup_j A_j \in \mathfrak{F}$.

A function $m(A)$ of sets $A \in \mathfrak{F}$ will be called a *measure* defined in E if the following conditions 1^0-3^0 are satisfied:

1^0) $0 \leq m(A) \leq +\infty$,

2^0) m is completely additive,

3^0) E is decomposed into countably infinite sets E_j of $\mathfrak{F}$ of finite $m(E_j)$:

$$E = \bigcup_j E_j, \quad m(E_j) < +\infty, \quad E_j \in \mathfrak{F} \,.$$

By 3^0), the space E itself is a set of $\mathfrak{F}$.

Sets belonging to $\mathfrak{F}$ will be called *measurable*, and the space E, together with its measure, will be called the *measure space* which will be denoted by $\mathfrak{E} = (E, m) = (E, \mathfrak{F}, m)$.

By means of the measure m, we can define an *outer measure* m^* defined for every subset of E in the usual way: