Fu, Z. and Li, Z.
Osaka J. Math.
41 (2004), 727–744

MEASURE-VALUED DIFFUSIONS AND STOCHASTIC EQUATIONS WITH POISSON PROCESS

ZONGFEI FU and ZENGHU LI

(Received January 7, 2003)

1. Introduction

Suppose that we are given a locally compact metric space E. Let $C(E)$ denote the set of bounded continuous functions on E, and $C_0(E)$ its subset of continuous functions vanishing at infinity. The subsets of non-negative elements of $C(E)$ and $C_0(E)$ are denoted respectively by $C^+(E)$ and $C_0^+(E)$. Let $(P_t)_{t\geq 0}$ be a strongly continuous conservative Feller semigroup on $C_0(E)$ with generator $(A, \mathcal{D}(A))$, where $\mathcal{D}_0(A) \subset C_0(E)$, and let $\mathcal{D}(A) = \mathcal{D}_0(A) \cup \{1\}$. Suppose in addition that $b(\,\cdot\,) \in C(E)$ and $c(\,\cdot\,) \in C^+(E)$ have continuous extensions to $\overline{E}$, the one point compactification of E, and that $c(\,\cdot\,)$ is bounded away from zero.

Let $M(E)$ be the space of finite Borel measures on E equipped with the topology of weak convergence. Let $W = C([0,\infty), M(E))$ be the space of all continuous paths $w\colon [0,\infty) \to M(E)$. Let $\tau_0(w) = \inf\{s > 0\colon w(s) = 0\}$ for $w \in W$ and let W_0 be the set of paths $w \in W$ satisfying $w(0) = w(t) = 0$ for all $t \geq \tau_0(w)$. We fix a metric on $M(E)$ which is compatible with its topology and endow W and W_0 with the topology of uniform convergence. Then for each $\mu \in M(E)$ there is a unique Borel probability measure $\boldsymbol{Q}_\mu$ on W such that for $f \in \mathcal{D}(A)$,

$$M_t(f) = w_t(f) - \mu(f) - \int_0^t w_s(Af - bf)\,ds, \quad t \geq 0, \tag{1.1}$$

under $\boldsymbol{Q}_\mu$ is a martingale with quadratic variation process

$$\langle M(f)\rangle_t = \int_0^t w_s\left(cf^2\right)\,ds, \quad t \geq 0, \tag{1.2}$$

where $\mu(f) = \int f\,d\mu$. The system $\{\boldsymbol{Q}_\mu\colon \mu \in M(E)\}$ defines a measure-valued diffusion, which is the well-known Dawson-Watanabe superprocess. In the sequel, we shall simply refer to it as a (A, b, c)-*superprocess*. We refer the reader to Dawson [1] and the references therein for the construction and basic properties of the process. A mod-

Research supported by NSFC (No. 10131040 and No. 10121101).