Kamae, T. and Yu-Mei, X.
Osaka J. Math.
41 (2004), 257–265

TWO DIMENSIONAL WORD WITH $2k$ MAXIMAL PATTERN COMPLEXITY

TETURO KAMAE and XUE YU-MEI

(Received October 7, 2002)

1. Introduction

For an infinite 1-dimensional word $\alpha = \alpha_0\alpha_1\alpha_2\cdots$ over a finite alphabet A, Teturo Kamae and Luca Zamboni [1] introduced the maximal pattern complexity as

$$p^*_\alpha(k) := \sup_\tau \sharp\{\alpha_{n+\tau(0)}\alpha_{n+\tau(1)}\cdots\alpha_{n+\tau(k-1)};\ n = 0,\ 1,\ 2,\dots\}$$

where the supremum is taken over all sequences of integers $0 = \tau(0) < \tau(1) < \cdots < \tau(k-1)$ of length k, and $\sharp S$ denotes the cardinality of the set S. They proved that α is eventually periodic if and only if $p^*_\alpha(k)$ is bounded in k, while otherwise, $p^*_\alpha(k) \geq 2k$ $(k = 1,\ 2,\dots)$.

Teturo Kamae, Rao Hui and Xue Yu-Mei [3] considered the maximal pattern complexity for 2-dimensional words defined on $\mathbb{Z}^2$ and proved that either $p^*_\alpha(k)$ is bounded in k or $p^*_\alpha(k) \geq 2k$ $(k = 1,\ 2,\dots)$ if α satisfies a 2-dimensional recurrence condition.

In this paper, we consider the maximal pattern complexity for 2-dimensional words defined on

$$\Omega := \mathbb{N}^2 \setminus \{(0,0)\}.$$

Let $\alpha = (\alpha(x,\,y)_{(x,y)\in\Omega}) \in A^\Omega$ be a 2-dimensional word over $\mathbf{A} = \{0, 1\}$ defined on Ω. Let τ be a finite set in $\mathbb{Z}^2$ with $(0,0) \in \tau$ and $\sharp\tau = k$, which is called a *k-window*. For any $i \in \Omega$ with $i + \tau \subset \Omega$, we denote

$$\alpha[i+\tau] := (\alpha(i+j))_{j\in\tau} \in A^\tau.$$

We also denote

$$\begin{aligned} F_\tau(\alpha) &:= \{(\alpha[i+\tau];\ i \in \Omega \text{ with } i+\tau \subset \Omega\} \\ p^*_\alpha(k) &:= \sup\{\sharp F_\alpha(\tau);\ \tau\colon k\text{-window}\} \quad (k = 1,\ 2,\dots). \end{aligned}$$

DEFINITION 1. α is called *eventually* 2-*periodic* if there exist $p,\ q \in \mathbb{Z}_+$ and $a,\ b \in \mathbb{N}$ such that for any $(x,\,y) \in \Omega$, $\alpha(x,\,y) = \alpha(x+p,\,y)$ holds if $x \geq a$ and $\alpha(x,\,y) = \alpha(x,\,y+q)$ holds if $y \geq b$.