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INTEGRAL GEOMETRY ON SUBMANIFOLDS OF DIMENSION ONE AND CODIMENSION ONE IN THE PRODUCT OF SPHERES

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1. Introduction

Let G be a Lie group and K a closed subgroup of G. Consider two submanifolds in a Riemannian homogeneous space G/K, one fixed and the other moving under g in G. Let the fixed one be M and the moving one be gN and let μ_G be the invariant measure on G. By taking the geometric invariant $\mathrm{vol}(M \cap gN)$, volume of the submanifold $M \cap gN$, and integrating with respect to $d\mu_G(g)$, we get so called the Poincaré formula. This can be briefly stated as follows.

Let M^p and N^q be submanifolds of dimensions p and q respectively, in a Riemannian homogeneous space G/K. Then many works in integral geometry have been concerned with computing integrals of the following form

$$\int_G \mathrm{vol}(M \cap gN)\, d\mu_G(g).$$

The Poincaré formula means equalities which represent the above integral by some geometric invariants of submanifolds M and N of G/K. For example in the case that G is the group of isometries of Euclidean space $\mathbf{R}^n$ and M and N are submanifolds of $\mathbf{R}^n$ then the result of above integral leads to formulas of Poincaré, Crofton and other integral geometers (see [6]). Especially R. Howard [1] obtained a Poincaré formula for Riemannian homogeneous spaces as follows:

Let M and N be submanifolds of G/K with $\dim M + \dim N = \dim(G/K)$. Assume that G is unimodular. Then

$$\int_G \sharp(M \cap gN)\, d\mu_G(g) = \iint_{M\times N} \sigma_K(T_x^\perp M, T_y^\perp N)\, d\mu_{M\times N}(x, y), \tag{1.1}$$

where $\sharp(X)$ denotes the number of elements in a set X and $\sigma_K(T_x^\perp M, T_y^\perp N)$ is defined by (2.1) in Section 2.

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