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LOCAL LIMIT THEOREM FOR RANDOM WALK IN PERIODIC ENVIRONMENT

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1. Preliminaries and Results

Let $(\Omega, \mathcal{F}, \boldsymbol{P})$ be a probability space on which all our random quantities will be defined. Let $\boldsymbol{Z}^d$ be the set of d-dimensional integer lattice. We consider Markov chain on $\boldsymbol{Z}^d$ with a transition function $P(x, y)$. We denote by $P_n(x, y)$ the n-th transition function of the Markov chain. We are interested in an asymptotic behaviour of $P_n(x, y)$ as $n \to \infty$, that is, a local limit theorem for the Markov chain. Spitzer showed a uniform estimate of a local limit theorem for random walk in $\boldsymbol{Z}^d$ (see, [10, Remark to P7.9 and P7.10]). The purpose of this paper is to extend his result to the Markov chain with the following assumptions.

ASSUMPTION 1.1. There exists $\boldsymbol{s} = (s_1, s_2, \ldots, s_d) \in \boldsymbol{Z}^d$ with $s_l > 0$, $1 \leq l \leq d$, such that

$$P(x + s_l\boldsymbol{e}_l, y + s_l\boldsymbol{e}_l) = P(x, y)$$

for every x, $y \in \boldsymbol{Z}^d$ and l, $1 \leq l \leq d$. Here $\boldsymbol{e}_l$, $1 \leq l \leq d$, denotes the basis vector $(\underbrace{0, \ldots, 0, 1}_{l}, 0, \ldots, 0)$ in $\boldsymbol{Z}^d$.

We call a Markov chain with this assumption *a random walk in periodic environment* (*RWPE* for abbreviation), and the vector $\boldsymbol{s}$ *period of RWPE*.

ASSUMPTION 1.2. The Markov chain is irreducible and aperiodic, that is, for every x, $y \in \boldsymbol{Z}^d$, there exists a positive integer $n_0(x, y)$ such that $P_n(x, y) > 0$ for all $n \geq n_0(x, y)$.

We set

$$\Xi = \{(j_1, j_2, \ldots, j_d) \in \boldsymbol{Z}^d \mid 0 \leq j_1 \leq s_1 - 1, \ldots, 0 \leq j_d \leq s_d - 1\}.$$

For $x \in \boldsymbol{Z}$ and l, $1 \leq l \leq d$, we denote by $T_l(x)$ the remainder obtained when x is divided by s_l, and put $T(x) = (T_1(x_1), T_2(x_2), \ldots, T_d(x_d))$ for $x = (x_1, x_2, \ldots, x_d) \in \boldsymbol{Z}^d$.