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CROSSING CHANGE AND EXCEPTIONAL DEHN SURGERY

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1. Introduction

Thurston's hyperbolic Dehn surgery theorem [11], [12] asserts that if a knot K in the 3-sphere S^3 is hyperbolic (i.e., $S^3 - K$ admits a complete hyperbolic structure of finite volume), then all but finitely many Dehn surgeries on K yield hyperbolic 3-manifolds. By an *exceptional surgery* on a hyperbolic knot we mean a nontrivial Dehn surgery producing a non-hyperbolic manifold. Refer to [3], [6] for a survey on Dehn surgery on knots. We empirically know that 'most' knots are hyperbolic and 'most' hyperbolic knots have no exceptional surgeries. In this paper, we demonstrate the abundance of hyperbolic knots with no exceptional surgeries by showing that every knot is 'close' to infinitely many such hyperbolic knots in terms of crossing change.

We regard that two knots are the same if they are isotopic in S^3. For a knot K in S^3, let $B_n(K)$ be the set of knots each of which is obtained by changing at most n crossings in a diagram of K.

Theorem 1.1. *For every knot K in S^3, $B_1(K)$ contains infinitely many hyperbolic knots each of which has no exceptional surgeries. In particular, an arbitrary knot can be deformed into a hyperbolic knot with no exceptional surgeries by a single crossing change.*

In Section 3, we raise some questions on the distribution of hyperbolic knots (with no exceptional surgeries).

2. Proofs

A (2-string) *tangle* is a pair (B, t) where B is a 3-ball and t is a pair of disjoint arcs properly embedded in B. We call (B, t) a *trivial tangle* if there is a homeomorphism from (B, t) to $(D \times I, \{x, y\} \times I)$, where D is a disk containing x and y in its interior. A tangle (B, t) is said to be *atoroidal* if $B - \operatorname{int} N(t)$ contains no incompressible tori. A tangle (B, t) is said to be *∂-irreducible* if $B - \operatorname{int} N(t)$ is ∂-irreducible.

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