

ON MARKOV CHAINS INDUCED FROM STOCK PROCESSES HAVING BARRIERS IN FINANCE MARKET

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1. Introduction

In Taiwan's market, there are lower and upper bounds on every day's stock price. The lower bound of today's stock price is defined by 93 % of the final price of yesterday's stock. And the upper bound of today's stock price is defined by 107 % of the final price of yesterday's stock. Under this background, we are interested in the effect of the lower bound and upper bound that cause every day's stock price in a long term. In words, what is the influence of the bounds on every day's stock price?

On the other hand, by the empirical studies [1] [2] [6], if the distribution of financial time series such as stocks returns are compared with the normal distribution, then fatter tails are observed. Besides, the standardized fourth moment for a normal distribution is 3 whereas for many financial time series a value well above 3 is observed by Mandelbrot [6]. Many other researchers [3] [4] [5] [7] [8] also report this feature and adopt the model with fat tail property to research financial problems. However, those researches above did not give the definite reasons of leading fat tails. Thus verifying the reason theoretically is a difficult problem but is a crucial research.

In order to research those problems in Taiwan's market, we use some kinds of diffusion processes $\{S_t\}_{t \geq 0}$ to drive the price of the stock. Furthermore we suppose that the stock price must be stopped at the bounds until the end of that day when the process hits the bounds. From this restriction to diffusions, we get a discrete Markov chain $\{X_n\}_{n \geq 0}$ in $(0, \infty)$. The rigorous definition of the Markov chain is given in the following section.

Due to the motivation above, we attempt to probe the relationship between the bounds (lower and upper) and the asymptotic behavior of $\{X_n\}_{n \geq 0}$. And if the invariant probability measure $\mu(\cdot)$ of $\{X_n\}_{n \geq 0}$ exists, we are interested in the tail of $\mu(\cdot)$.

Therefore, the purpose of this paper is to research the (positive) recurrence and transience of $\{X_n\}_{n \geq 0}$. Also we compare the tail of the invariant probability measure of $\{X_n\}_{n \geq 0}$ with $\{S_t\}_{t \geq 0}$. Our results imply that if $\{S_t\}_{t \geq 0}$ is recurrent and the bounds satisfy some conditions, then the effect of lower and upper bounds gives a phenomenon of fat tails. Indeed, in other countries, the governments also give a restriction on stock processes when stock market falls down. But the restriction is not so clear as Taiwan's market. Also the restriction is sometimes ambiguous and is difficult to de-