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GLOBAL WEAK SOLUTIONS OF THE NAVIER-STOKES EQUATIONS FOR MULTIDIMENTIONAL COMPRESSIBLE FLOW SUBJECT TO LARGE EXTERNAL POTENTIAL FORCES

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1. Introduction

1.1. Background In this paper, we are concerned with compressible, viscous, isentropic flow in three (and two) space dimensions. The fluid motion is described in the following form by the conservation laws of mass and momentum:

$$\rho_t + \mathrm{div}(\rho u) = 0, \tag{1.1}$$

$$(\rho u)_t + \mathrm{div}(\rho u \otimes u) + \nabla p(\rho) - \mu\Delta u - (\lambda + \mu)\nabla(\mathrm{div}\, u) = \rho f. \tag{1.2}$$

Here $t \geq 0$ is time, $x = (x_1, \ldots, x_n) \in \mathbf{R}^n$ ($n = 2$ or 3) is the spatial coordinate,

$$\rho(x,t),\ \ u(x,t) = (u^1(x,t), \ldots, u^n(x,t)),\ \ p(\rho) = a\rho^\gamma\ \ (a > 0,\ \ \gamma \geq 1)$$

represent respectively the fluid density, velocity, and pressure, $f(x) = (f^1(x), \ldots, f^n(x))$ is the external force, and μ, λ are viscous coefficients which satisfies $\mu > 0$, $3\lambda + 2\mu \geq 0$ by physical requests.

The local (in time) solvability to the various initial boundary value problems for the full Navier-Stokes equations (which include also the conservation law of energy) was obtained by Nash [11], Solonnikov [14], and Tani [16]. The first result about the global theory is that of Matsumura and Nishida [7], who proved the global existence of H^3-solutions around a constant state for the Cauchy problem without external forces. Afterwards, in the case that external potential force field is small enough, and for the interior or exterior problems, almost the same results were derived by Matsumura and Nishida [8]–[9], and Valli [18]. But there have been no remarkable results in the case with *large* external potential forces except for that of Matsumura and Padula [10], who proved the stability of the corresponding stationary state (more precisely, the global existence of H^3-solutions which tend toward the stationary solution) for the interior problems.

On the other hand, discontinuous, namely weak solutions play an important role in the physical as well as in the mathematical theory, and the problem of global exis-