Mori, M.
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# DYNAMICAL SYSTEMS ON FRACTALS IN A PLANE

MAKOTO MORI

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## 1. Introduction

We considered in [3] ergodic properties of one-dimensional piecewise linear mappings, and solve the spectral problem of the Perron-Frobenius operator associated with these mappings. The main tool to solve it is a renewal equation on a signed symbolic dynamics. By this renewal equation, we define a matrix which we call the Fredholm matrix, and the spectral problem of the Perron-Frobenius operator and also the dynamical zeta function are characterized by this matrix.

Extending this idea to Cantor sets in the unit interval, we considered in [5] and [6] the ergodic properties of the dynamical systems on them. Also extending the idea of signed symbolic dynamics in one-dimensional dynamical system, which express the orbits of endpoints of subintervals of monotonicity, we introduced signed symbolic dynamics on a plane in [4], which corresponds to vertices and edges of polygons, and studied dynamical systems on it.

In this article, combining these ideas, we will study the Hausdorff dimension of Cantor sets in a plane. We will consider two types of Cantor sets which is generated by Koch-like mappings and Sierpinskii-like mappings (definition will be given in §3). As in 1-dimensional mappings, we construct the $\alpha$-Fredholm matrix $\Phi(z\colon \alpha)$, and take $\alpha_0$ the maximal solution of $\det(I - \Phi(1\colon \alpha)) = 0$, and put $d_\Phi = 2\alpha_0$. The theorem which we will prove in this paper is the following:

**Theorem 1.** *Let $F$ be a Koch-like mapping or a Sierpinskii-like mapping. Assume that $d_\Phi/2$ is the simple zero of $\det(I - \Phi(1\colon \alpha))$, and $\xi d_\Phi/2 - \nu > 0$. Then the Hausdorff dimension of the Cantor set generated by $F$ equals $d_\Phi$.*

The numbers $\xi$, which we call the lower Lyapunov number, and $\nu$ are defined by

$$\xi = \liminf_{n\to\infty} \operatorname*{ess\,inf}_{x\in I} \frac{1}{n} \log|\det D(F^n)(x)|,$$

and

$$\nu = \limsup_{n\to\infty} \sup_{l} \frac{1}{n} \log \#\{w\colon |w| = n, \langle w\rangle \cap l \neq \emptyset\},$$