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TOPOLOGICAL ENTROPY FOR DIFFERENTIABLE MAPS OF INTERVALS

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Let I be a compact interval of the real line. For a continuous map $f : I \to I$ by Misiurewicz et al. ([1, 12, 13]) the following relation between the topological entropy $h(f)$ and the growth rate of the number of periodic points is known:

$$(*) \qquad h(f) \leq \limsup_{n\to\infty} \frac{1}{n} \log \sharp \operatorname{Per}(f, n)$$

where $\operatorname{Per}(f, n)$ denotes the set of all fixed points of f^n for $n \geq 1$, and $\sharp A$ the number of elements of a set A. (The equality of the expression $(*)$ does not hold in general. For instance, the topological entropy of the identity map is zero, nevertheless all of points of the interval are fixed by this map.)

For a periodic point p of f with period n we put

$$\mathcal{O}_f^+(p) = \{p, f(p), \cdots, f^{n-1}(p)\}.$$

Then we say that q is a *homoclinic point* of p if $q \notin \mathcal{O}_f^+(p)$ and there are a positive integer m with $f^m(q) = p$ and a sequence $q_0, q_1, \ldots, q_k, \ldots \in I$ with $q_0 = q$ such that

$$f(q_k) = q_{k-1} \ (k \geq 1), \qquad \lim_{k\to\infty} |q_k - \mathcal{O}_f^+(p)| = 0$$

where $|x - A| = \inf\{|x - y| : y \in A\}$ for $x \in I$, $A \subset I$. It is known by Block ([2, 3]) that $h(f)$ is positive if and only if f has a homoclinic point of a periodic point.

In this paper we shall establish more results (Theorems 1 and 2) for differentiable maps of intervals. To describe them we need some notations.

Let $f : I \to I$ be a $C^{1+\alpha}$ map ($\alpha > 0$). A periodic point p of f with period n is a *source* if

$$\nu(p) = |(f^n)'(p)|^{1/n} > 1.$$

For $n \geq 1$, $\nu > 1$ and $\delta > 0$ we define an f-invariant set by

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